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THE NEW MATH AS AN INTERNATIONAL PHENOMENON[1]

A "NEW MATH" COMO UM FENÔMENO INTERNACIONAL

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ABSTRACT

The new math was a several-decade-long movement to update school mathematics that began in the mid twentieth century in many countries around the world. It took many forms, but much of it involved the preparationof new instructional materials, including textbooks. Much of the new math activity in Europe and North America was stimulated by conferences and seminars of the Organisation for Economic Cooperation and Development (OECD) in the early 1960s. Although the initial reform efforts addressed secondary school mathematics courses that prepared pupils for tertiary education, those efforts quickly spread to primary school mathematics, to pupils not headedto tertiary education, and to non-OECD countries. Mathematicians and schoolteachers were divided about the direction that the new math reforms took. Although thenew math is often deemed a failed endeavor, it changed notonly school mathematics but also the way people and countries viewed school mathematics.
Keywords: Mathematics Teaching. Reform Movement. International Impact. Movement Evaluation.

RESUMO

A "New Math" foi um movimento de várias décadas para atualizar a matemática escolar que começou em meados do século XX em muitos países ao redor do mundo. Assumiu muitas formas, e muitas delas envolveu a preparação de novos materiais instrucionais, incluindo livros didáticos. Grande parte da nova atividade matemática na Europa e na América do Norte foi estimulada por conferências e seminários da Organização para Cooperação e Desenvolvimento Econômico (OCDE) no início dos anos 1960. Embora os esforços iniciais de reforma abordassem os cursos de matemática do ensino secundário que preparavam os alunos para o ensino superior, esses esforços rapidamente se espalharam para a matemática do ensino primário, para alunos que não frequentavam o ensino superior e para países não pertencentes à OCDE. Matemáticos e professores estavam divididos sobre a direção que as novas reformas matemáticas tomaram. Embora a matemática moderna seja muitas vezes considerada um movimento fracassado, ela mudou não apenas a matemática escolar, mas também a maneira como as pessoas e os países viam a matemática escolar.
Palavras-chave: Ensino da Matemática. Movimento de Reforma. Impacto Internacional. Avaliação do Movimento.

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INTRODUCTION

From the mid-1950s to the mid-1970s, and later in some places, a variety of attempts to change school mathematics were made under the label "modern mathematics" or "the new math." These efforts arose from various sources and took many forms, but they tended to have in common a desire to bring school mathematics closer to the academic mathematics of the twentieth century—to eliminate inane jargon and make it better preparation for the mathematics being taught in the university (Kilpatrick 1997/2009). An important influence in this regard was the work of the Bourbaki group, which beginning in the 1930s attempted to generalize, formalize, and unify all of pure mathematics (Dieudonné 1970; Halmos 1957; Moon 1986; S. Roberts 2006).

After the Soviet Union launched its first Sputnik into earth orbit in October 1957, efforts to modernize school mathematics increased on both sides of the Atlantic. An especially noteworthy event was a 2-week seminar organized by the Organization for European Economic Cooperation (OEEC; later joined by nations outside Europe to form the Organisation for Economic Cooperation and Development, OECD) and held from 23 November to 4 December 1959 at the Cercle Culturel de Royaumont, Asnières-sur-Oise, France. The seminar addressed a variety of proposals for modernizing the school mathematics curriculum, the teaching of mathematics, and the preparation of teachers. Jean Dieudonné, a former member of the Bourbaki group who represented what the seminar report (OECD 1961a) itself called "an extremist point of view" (p. 31), led the program section on new thinking in mathematics. He challenged the participants to remove Euclidean geometry from its central position in the secondary school curriculum, saying: 'À *bas Euclide! Mort aux triangles!* [Down with Euclid! Death to triangles!]'' (S. Roberts 2006, p. 157). Instead, Dieudonné said that students entering the university should, on one hand, be familiar with a certain number of elementary techniques such as elementary linear algebra, analytic trigonometry and some calculus. On the other hand, the student should already be fairly well trained in the use of logical deduction and have some idea of the axiomatic method. (OECD 1961a, p. 32)

He wanted the "new language" (p. 34) that mathematics had acquired in the twentieth century—the language of sets, mappings, groups, vector spaces, and so on—to be adopted by school mathematics in the same way it had been adopted by Bourbaki (1970/2006). Dieudonné was careful to say that in seeming to be highly critical of geometry, he did not want "to minimise its importance" (OECD 1961a, p. 45). Others, however, interpreted his battle cry to mean that geometry should be removed altogether from secondary school mathematics (S. Roberts 2006, ch. 8), and in some cases, it essentially was.

After Royaumont, efforts began in many of the OECD countries to reform school mathematics along the lines proposed at the conference. In August and September 1960 in Dubrovnik, Yugoslavia, in response to a resolution at Royaumont, the OECD convened a "Group of Experts [to prepare] a detailed synopsis for modern treatments of the mathematical curriculum in European secondary schools" (OECD 1961b, p. 3). The resulting document addressed only part of the curriculum (i.e., algebra, geometry, and statistics) and only part of the student body (i.e., ''the upper fifty percent of students in the European Lycée or Gymnasium" [p. 3]), but it, too, stimulated OECD countries to move ahead with their reform efforts. For example, after a 1961 conference at Southampton University (Cooper, 1985; Thwaites 1961), the School Mathematics Project (SMP) was organized to develop textbook materials intended originally for secondary schools in England. The materials were later to be adapted for use throughout the English-speaking world. Simultaneously, the Midlands Mathematics Experiment, a parallel but more modest effort, also began work in the United Kingdom (Cooper 1985). In the Netherlands, a commission (CMLW) was established in 1961 to modernize the secondary school mathematics curriculum (Moon 1986, pp. 72–75). Rather than developing textbook or other instructional materials, however, the CMLW undertook efforts in teacher retraining. In Belgium, the first of six groundbreaking volumes entitled *Mathématique Moderne* by Georges and Fréderique Papy was published in 1963. In the proceedings of an OECD international working session on new teaching methods for school mathematics in November that same year in Athens, Greece (Fehr 1964), there were reports on ''innovations in mathematics education since 1960'' (pp. 310–369) from the 20 founding OECD countries: Austria, Belgium, Canada, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States of America. Notable innovations were being made in some of these countries; others had yet to begin any efforts to innovate. An influential report from the National Advisory Committee on Mathematical Education (1975) in the United States, surveying US reform efforts and arguing against dichotomous choices that were being proposed for school mathematics (e.g., skills vs. concepts; intuition vs. formalism; old vs. new) recommended "that the term 'new math' be limited in its use to describe the multitude of MATHEMATICS education concerns and developments of the period 1955–1975" (p. 137). In other words, the term is "a label not so much for a cohesive set of reform proposals and activities as for an era during which a variety of reforms were undertaken''

(Stanic and Kilpatrick 1992, p. 413). That observation can also be made about the new math as it was implemented in other countries around the world.

1. BEGINNINGS

Even before 1957 and Sputnik, efforts were being made in the United States to reform secondary school mathematics. Near the end of 1951, a committee of University of Illinois faculty in engineering, mathematics, and education issued a report arguing that beginning engineering students at the university needed increased preparation in high school mathematics (UICSM Project Staff 1956/1970). The next year, recognizing that the problem went beyond engineering and extended throughout the state, another committee (the University of Illinois Committee on School Mathematics [UICSM]) with much the same composition but including representatives from the University High School, the laboratory school of the College of Education, began the development of a secondary school mathematics curriculum. The resulting materials, a complete curriculum for Grades 9 through 12 supported by the Carnegie Corporation and the US Office of Education, were guided by two principles: the language used should be precise, and the student should have the opportunity to discover generalizations (Beberman 1958). In addition to UICSM, groups of mathematicians and educators at other institutions— such as Ball State Teachers College, the University of Maryland, Southern Illinois University, and Boston College—undertook similar projects to reform secondary school mathematics during the 1950s (Wooton 1965, p. 9).

In 1955, the US College Entrance Examination Board (CEEB) appointed a Commission on Mathematics whose charge was to study the mathematics needed by students entering the university. The Board's examiners were concerned about a growing gulf between their examinations and the mathematics being taught in some college preparatory programs as well as about the low levels of mathematical understanding and poor attitudes toward mathematics on the part of many high school graduates (Meder 1959; Wooton 1965, p. 8). The commission's influential final report was accompanied by a series of appendices detailing a proposed secondary mathematics curriculum (Commission on Mathematics 1959a, b).

> The commission recommended the introduction ofnew topics such as logic, modern algebra, probability,and statistics so that secondary school curricula would better reflect important new facets of pure and applied mathematics. It also proposed the combination of previously separate courses in plane and solid geometry and in advanced algebra and trigonometry so that students

> could proceed more quickly to the frontiers of mathematics and meet the national need for a sophisticated scientific workforce. (Fey and Graeber 2003, p. 524)

The need to improve the mathematical preparation of that workforce—and in particular, the need to recruit and prepare an adequate number of mathematicians—was the motivation behind two conferences of mathematicians held in early 1958 that led to the establishment of the School Mathematics Study Group (SMSG) financed by the National Science Foundation and under the leadership of Edward G. Begle, then of Yale University (Wooton 1965, pp. 9–16). SMSG ultimately became the largest and best known of the US mathematics curriculum development projects.

Similar efforts to examine school mathematics were underway in Europe, although no curriculum projects had yet been launched. For example, the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), which was founded by Caleb Gattegno in 1950, held periodic meetings that brought together pedagogues, psychologists, epistemologists, mathematicians, philosophers, and secondary school teachers in efforts to improve mathematics teaching (Furinghetti, Menghini, Arzarello, and Giacardi 2008). In the first book by the CIEAEM founders (Piaget et al. 1955), André Lichnerowicz argued "for an experimental effort to introduce something of the spirit of modern mathematics into arithmetic, algebra, elementary geometry, by pointing out the structures of ring, group, and vector space and by adapting the language" (Furinghetti et al. 2008, p. 135). Another effort in Europe occurred when "as early as 1950 a joint conference between UNESCO and the International Bureau of Education considered amongst other topics 'mathematics in primary schools'" (Moon 1986, p. 47; see also Barrantes and Ruiz 1998, ch. 1, para. 3). Moon questions the claim that "a 'wave' of development in the USA 'crossed over' to Europe" (p. 46), arguing instead that the preface to the 1959 Royaumont conference report (OECD 1961a) shows that "a pattern of 'parallel' innovation" (p. 46) would be a more accurate description.

2. EXTENSIONS

After its beginnings in a few OECD countries with projects primarily aimed at the reform of secondary school mathematics for pupils headed to tertiary education, the new math soon began to spread in three directions: (a) to primary school mathematics, (b) to become a curriculum for all pupils, and (c) to countries outside the original OECD.

2. 1. Primary school mathematics

Already in the reports at the OECD working session in Athens in 1963, the United Kingdom noted that the Nuffield Foundation was starting a project to reform school mathematics for ages 5–13 (Fehr 1964, pp. 365). The United States reported that the SMSG project was starting work on elementary school mathematics (p. 366) and that there were "major movements going on in the elementary school program that now promise better prepared entrants to the secondary school" (p. 268). In 1965, Heinrich Bauersfeld was appointed to direct a primary school mathematics project in Frankfurt, Germany, and the project—the *Alef Programm*—was launched the next year (Moon 1986, p. 161; see also Bauersfeld, 1972). Bauersfeld wanted *Alef*, unlike some of the projects in other countries with which he was familiar, to move away from "reform of the mathematical content of the elementary school curriculum [and, instead,] to focus on social learning and language development" (Moon 1986, p. 166).

2. 2. Mathematics for all

In addition to spreading to lower grades, efforts to reform school mathematics also began to be addressed to pupils who might not be among those intending or even eligible to pursue tertiary education. Countries such as France and the United Kingdom were establishing comprehensive schools during the 1960s, and revised textbook materials were needed for the pupils in those schools. In the United Kingdom in 1967, the Schools Council undertook the Mathematics for the Majority Project (MMP; Kaner 1973; Wood 1975), which was aimed at pupils of "average and below average ability" (Howson 1978, p. 204). The MMP,

> like the Nuffield Mathematics Project, had to consider not only content but also teaching methods, for the target audience presented particular problems. Also the teachers addressed by the project had, in general, never specialised in mathematics and often only taught the subject part-time: the less-able students too frequently were given the less-able teachers. (p. 205)

Eventually, in 1971, the Schools Council established a "Continuation Project" (Kaner 1971) to prepare pupils' materials for the MMP.

Many new math reformers believed that the same (improved) mathematics should be taught to all students. "At the height of that [new math] movement it was commonly assumed that the new mathematics was a mathematics for all" (Damerow and Westbury 1984, p. 22). The textbooks prepared by the SMSG, for example, for Grades K through 6 were intended to be used by all students. SMSG had a panel whose major task was to prepare suitable mathematics texts for non-college-bound students in Grades 7 and 8 ("Introduction to Secondary School Mathematics") and in Grade 9 ("Introduction to Algebra"). The assumption was that by slowing the texts down—taking two years rather than one—the same material could be learned by less-advanced and less-well-prepared students (Begle 1971; Walmsley 2003). In the United Kingdom, in contrast, differentiation of the mathematics taught according to the pupils' perceived "ability" appeared somewhat more entrenched, at least in the minds of the reformers. Discussing the presumably progressive views of members of the Association for Teaching Aids in Mathematics (ATAM, later the ATM or Association of Teachers of Mathematics), Barry Cooper (1985) had

> the overall impression … of a commitment amongst members to a mathematics-for-all, as opposed to a mathematics-for-some and arithmetic-for-others, approach, but with the meaning of 'mathematics' varying, and the degree of commitment wavering, according to members' particular locations, current occupational problems and reference groups. (p. 78)

2. 3. Non-OECD countries

In 1978, the International Commission on Mathematical Instruction (ICMI) published a report, '"Change in Mathematics Education since the Late 1950's" (Freudenthal 1978a), in which mathematics educators reported on developments in their 16 countries, the great majority outside the original OECD: Australia, Bangladesh, France, Great Britain, Hungary, India, Iran, the Netherlands, Nigeria, Poland, Sierra Leone, Sri Lanka, Sudan, Thailand, the United States, and the West Indies. Efforts to change the school mathematics curriculum had been undertaken in the preceding two decades in every country for which a report was made, but those efforts had met with varying degrees of success, went in different directions, and were affected by other changes in society and the educational system.

In the Soviet Union, curriculum reform efforts began in the early 1960s and were led by the noted mathematician Andrey Kolmogorov, whose team produced a new mathematics curriculum that was approved in 1968 (Abramov 2010, p. 97). Many of the reforms resembled those being undertaken by OECD countries. In the early grades, "Arithmetic" was eliminated as a separate subject, becoming '"Mathematics" and including preparatory materials for the study of algebra and geometry; some topics usually studied later, such as complex numbers and elementary probability theory, were eliminated from the program; and other topics, such as elements of calculus, geometric transformations, vectors, and coordinates, were added.

> Students also were to be given a substantive introduction to the axiomatic method. All of these [new topics] served the central aim of the curriculum, which was to enrich the course in mathematics with ideas that had become significant in an age of accelerating scientific

> technological progress as elements of a common culture. Another important goal was to increase the logical purity of the exposition. (p. 99)

The new Soviet mathematics curriculum included some attention to elementary set theory, but not nearly as extensively as in the reforms taking place in the OECD countries. Kolmogorov was primarily concerned with introducing a "more precise and complete system of notation and exposition for mathematical texts" (p. 99) so as to prepare pupils for coming changes in information technology.

In some countries, reform efforts included attempts to import new math instructional materials from other countries. The Entebbe Project (Williams 1971), for example, also known as the African Mathematics Programme (Ohuche 1978), was a project of the US Education Development Center that brought curriculum developers from the United States, and a few from the United Kingdom or who had been educated there, together with participants from Ghana, Ethiopia, Liberia, Kenya, Malawi, Nigeria, Uganda, Sierra Leone, Tanzania, and Zambia to work as writing teams in summer workshops annually from 1962 to 1968 to develop mathematics textbooks, teachers guides, and other instructional materials. The project's name came from the city— Entebbe, Uganda—where the first three of those summer workshops were held. "By 1971, the project had published sixty-seven different textbooks and teacher guides; by some estimates, 2 million African pupils used an 'Entebbe Math' book or a translation of one" (Zimmerman 2011, p. 11). The textbooks were heavily influenced by the views and approaches of the US and the expatriate UK participants. At the first workshop, in July 1962,

> the American participants arrived with SMSG in their pockets…. Ten African countries began trials of the new math. In turn, the British [expatriates] began their own writing projects based on their [familiarity with] SMP [but ultimately adapted to African students and their background]: the Joint Mathematics Project (JMP), was begun in West Africa and the East African School Mathematics Project (EASMP) in East Africa. (Swetz 2009, p. 34)

Unfortunately, the African teachers who would use those textbooks were not well equipped to teach the new material. Zimmerman (2011) documented the ineffectiveness of the so-called refresher courses for teachers in Ghana during the 1960s that were designed to acquaint them with the modernized content.

A similar effort to import ideas and materials from other countries was made by UNESCO when it was asked by the Arab States in 1966 to assist in improving mathematics education (Jacobsen 1996). "Seminars, syllabus determination, textbook writing sessions, and training sessions for teachers were held, which resulted in a new mathematics course for secondary schools which was implemented in most schools in the Arab States" (p. 1246). As Malaty (1999) noted, 8 of the 22 authors of the textbooks for senior secondary school were from "outside the Arab world"(p. 238)—mostly the United States and the United Kingdom—and the books were written in English and then translated into Arabic.

Except for the JMP and EASMP projects, in which the textbooks were written by teachers resident in the countries concerned, the well-intentioned efforts of reformers in OECD countries to transfer their efforts to non-OECD countries did not work out as planned.

> Even when attempts were made to produce original materials specifically for the countries concerned, forexample, the Entebbe Project and the UNESCO Arab States Project, the writing teams were dominated mathematically and professionally, if not numerically, by Western authors who lacked any prior understanding of the educational systems of the countries concerned and, more importantly, of the social ethos that was manifested in the schools. (Howson, Keitel, and Kilpatrick 1981/2008, p. 246)

Begle (1969) summarized the emerging recognition among the new math reformers that their ideas and materials could not be easily transplanted across national boundaries:

> Many countries are asking not only the U.S., but also others of the affluent countries, for assistance in improving their mathematics education programs. Having looked into a number of attempts to honor these requests, I am convinced that failure to studythe cultural milieu of the proposed reforms has often resulted in a serious waste of time, effort and money. (p. 241)

3. REACTION

Opposition to the new math movement in the United States began even before most of the curriculum projects had begun their work. Morris Kline, who was to become the most prominent antagonist of US new math efforts, published an article in the October 1958 *Mathematics Teacher* based on a presentation he had made to the National Council of Teachers of Mathematics. In it, he argued against what he understood to be the position of the CEEB Commission on Mathematics; namely, that traditional mathematics was outmoded and that the abstract should be taught before the concrete. In a reply to Kline, Albert Meder (1958), who served on the Commission, argued that Kline had misinterpreted the Commission's position and should wait until its final report appeared before attacking it. Undeterred, Kline (1961) launched an attack under the headline "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress," and the "math wars" (DeMott 1962) had begun.

In 1962, a memorandum "On the Mathematics Curriculum of the High School" was authored by Kline and Lipman Bers of New York University and George Pólya and Max Schiffer of Stanford University (see D. L. Roberts 2004). Signed by the 4 authors and an additional 61 US and Canadian mathematicians, the memorandum was published simultaneously in the American Mathematical Monthly and the Mathematics Teacher. It reiterated many of the points that Kline had made in earlier articles, arguing that the curriculum should cater to the needs of all students and not just those who might become mathematicians, that students should not be introduced to abstractions prematurely, that they should see the links between mathematics and the other sciences, that intuitions and conjectures should come before formal proof, that wherever possible mathematical ideas should be introduced as they had arisen genetically, and that traditional school mathematics should not be entirely replaced by so-called modern mathematics. The memorandum was followed 3 years later by a set of essays for and against the new math (Moise, Calandra, Davis, Kline, and Bacon 1965) and 11 years later by what was to prove the final shot in the US math wars: Kline's (1973) book Why Johnny Can't Add: The Failure of the New Math. Despite the title's reference to arithmetic, the book was actually aimed at the efforts to reform secondary school mathematics. As Malaty (1999) noted, the criticisms of the new math by Kline and others in the United States were similar to those that would appear in other Western countries, as was the resulting public opinion. The difference was that the US criticisms came first and were ''more evident'' (p. 233).

Not all the reaction against the US new math efforts arose from outside the phalanx of reformers. Max Beberman (1958), who headed the UICSM project, saw the movement of the new math from the secondary grades to the elementary grades in the United States as "hasty and unwise," saying, "We're in danger of raising a generation of kids who can't do computational arithmetic" (quoted in Schwartz 1964, p. 1). He claimed that elementary school teachers were frightened by the "esoteric mathematics" (p. 22) they were being expected to teach. "Changes in the elementary school program must be made very carefully and slowly, because large numbers of teachers must be retrained" (p. 22). Given that they had to teach a variety of subjects, elementary school teachers could not be expected to change the mathematics they taught as rapidly as teachers who specialized in the subject.

In France, the new math reforms of school curricula were initiated in 1966 with the appointment of the Lichnerowicz Commission (headed by the eminent mathematician Andre´ Lichnerowicz) to plan the changes (D'Enfert and Gispert 2011; Gispert and Schubring 2011). Gispert (2009) noted that the commission claimed that mathematics was "a deductive science and not an experimental one"(p. 46) and, for example, that pupils were taught affine geometry before Euclidean geometry even though "affine geometry is … the most irrelevant model of the real world and every day life" (p. 46). As the commission began its work, there was debate "but little opposition to reform" (Moon 1986, p. 105). When the ministry of education committed to a modernized mathematics program in early 1970, however, the level of debate increased and opposition appeared. In 1971, the equally eminent René Thom (a member of Bourbaki, whereas Lichnerowicz, although he helped found the group, was never a member) published, in the *American Scientist,* an article questioning the efforts to update the French curriculum, pointing to the confusion and concern being felt by many parents and teachers over the new terminology. Dieudonné (1973) replied, "We cannot afford to let future managers and technicians spend most of the precious years of school life absorbing useless knowledge taught by obsolete methods" (p. 19). He argued that after a generation had passed, parents would no longer be anxious and, with the help of professional mathematicians, educators might be able to engage in "sensible teaching of mathematics from kindergarten to graduate school" (p. 19).

In the Soviet Union, even though the Kolmogorov reform was still underway, a counter-reform began in 1978 that was led by mathematicians from the USSR Academy of Sciences (Abramov 2010). The major point of conflict appeared to be that the college entrance examinations in mathematics had not been modernized along with the curriculum, and some mathematicians resisted the idea of changing the exams. Rather than addressing some of the admitted flaws in the Kolmogorov-inspired curriculum and textbooks, the counter-reformers managed to convince the authorities to develop a new curriculum and new textbooks. Eventually the dispute became entangled in Communist Party ideology, which clinched the case favoring the critics of reform.

Some countries seem to have missed much of the new math movement even as international contacts increased between countries. For example, the Japan Society of Mathematical Education organized some 15 visits to foreign countries between 1966 and 1981 (Sekiguchi 2000, p. 111); Japan participated in the international studies of mathematics conducted by the International Association for the Evaluation of Educational Achievement that began in 1964; and it participated in the series of International Congresses of Mathematical Education that began in 1969. Despite its growing international outlook, however, the greatest involvement Japan had in new math activities, according to Sekiguchi, seems to have been a 1964 study seminar in Tokyo and Kyoto in which two mathematicians from the SMSG project informed over 100 Japanese mathematics educators about their work and a 1971 US–Japan seminar on mathematics education that brought together 9 educators from the United States and 20 from Japan to discuss new math "goals, curriculum, [and] teaching method" (p. 111). In contrast, some authors (Suzuki, Kuroda, and Li 2000; Suzuki and Yanagimoto 2008) have claimed that in the mid-1960s and early 1970s, the Japanese ministry of education attempted a top-down version of the new math that later “wrecked on a reef” (Suzuki et al. 2000). Either way, the resulting impact on Japanese school mathematics practice seems to have been minimal.

Another example of little or no influence is the Netherlands (Freudenthal 1978b), where at the secondary school level the new math “never became a public affair nor a target for frustrated teachers or hostile parents” (p. 265), and education at the elementary school level “never fell a victim to the slogan and fashion” (p. 267) of the new math.

Freudenthal (1979) characterized the new math as

> a fierce competition between both experts and charlatans with, in general, distressing results. Those who could not catch up were the poor people in the classroom who were expected to teach and to learn New Math, which most often was rather New Nonsense—unteachable, unlearnable, unmathematical. (p. 321)

4. LEGACY

The standard verdict regarding the new math is captured in the subtitle of Kline’s (1973) book: it failed. Here are some common reasons given for the so-called failure:

> Why did the “new math” fail? … It was a top-down reform, initiated by the mathematical community, without buy-in from teachers or the public…. One cause of failure of the “new math” was that it represented an absolutist philosophy in what was emerging as a fallibilist world….
>
> The “new math” curriculum changes were not accompanied by larger systemic changes, and were thus judged as failures. (Davidson and Mitchell 2008, pp. 147–148).

That last reason is among the most powerful. As Damerow and Westbury (1984) observed:

> The «new mathematics» showed decisively how problematic major change in a subject can be: while there were some admirable experiments which showed what might have been done in elementary schools under the name of new mathematics, «experimental» outcomes were not generalized to school systems as wholes. (p. 22)

Failure, however, seems much too simple a diagnosis. Robert B. Davis (2003) observed that the following claims about the new math were often made:

- There was once such a thing as *the* “new math”;
- It was widely adopted across the United States;
- But, unfortunately, it failed.

In actual fact, every one of these three statements is false. (p. 625) Davis made, instead, the following claims:

- Many alternative school mathematics programs were created and implemented; there was no single alternative program that could be labeled *the* new math, as if it were a recognizable well-defined thing.
- Most schools were affected relatively little.
- In places where some of the new programs were implemented very carefully, by people who understood them, the results were often dramatic—dramatically better, that is, not dramatically worse! In no way should this be described as '"failure". (pp. 626–627)

It should be noted that Davis's second point above, although true for the United States and the United Kingdom, does not apply to countries with a national curriculum such as France and Russia. In those countries, every teacher, learner, and school was necessarily involved in reform efforts.

In no country did school mathematics return to where it had been before the new math movement began: The pendulum is not a suitable metaphor for curriculum change. Certainly, various unsuccessful features of the new math did not survive. Number systems with bases other than 10, for example, introduced to help children see contrasts with features of the decimal system, proved confusing and tedious, especially when children were expected to memorize calculations in those systems. Basing an introduction to algebra on axioms, definitions, and theorems proved unwieldy for teachers and pupils to handle, as did axiomatic set theory. In some new math programs, the vocabulary load became overwhelming, and so unproductive terminology had to be eliminated.

On the other hand, many of the ideas brought into school mathematics by the new math have remained. For example, textbooks still refer to sets of numbers and sets of points. Much of Euclidean geometry has be replaced or overlaid by ideas from transformation geometry, coordinate geometry, and vector geometry, with coordinate geometry appearing early. Pupils encounter and solve inequalities along with equations. Numbers are organized into systems that have properties, such as the distributive property, that can help simplify computations. Simple ideas from statistics and probability are introduced in the early grades. Terms such as *number*, *numeral, unknown, inverse, relation, function,* and *graph* are given reasonably precise definitions and used to clarify notions of quantity, space, and relationships.

Before the new math era, no one thought of school mathematics as something to be reformed or updated; it simply was what it was. The new math reformers knew almost nothing about the school mathematics curriculum in other countries or, in some cases, in their own country. By the time the new math era ended, in contrast, everyone concerned with school mathematics had a much better sense of what was going on around the world.

Many new math reformers began their efforts with the view that the curriculum would be brought up to date mathematically if they could simply get their new syllabuses and textbooks into the hands of students and teachers. By the end of the era, they had come to see that much more was required. At the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it with his or her pupils—all situated in a specific educational and cultural context.

From a distance, school mathematics looks much the same everywhere. Countries include many of the same topics in their syllabuses and expect pupils to solve many of the same sorts of problems. International comparative studies are predicated on having a common framework on which to base the assessment of mathematics achievement. Up close, however, each country has a unique school mathematics—a product of its history, culture, and traditions, and conforming to its social, political, and educational systems. Instructional materials and practices in school mathematics cannot be transported across borders as if they were a common currency. The new math era taught us the paradox of curriculum change: The more school mathematics is internationalized, the more clearly its national character is revealed.

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