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Geometric Transformations and Modern Mathematics: What do Brazilian Curricular guidelines say between the 1950s to 1970s?

Transformações Geométricas e Matemática Moderna: o que dizem as normativas curriculares brasileiras no período de 1950 a 1970?

Transformations Géométriques et Mathématiques Modernes : Que disent les programmes brésiliens entre 1950 et 1970 ?

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ABSTRACT

This research is part of a larger project examining the relationship between the History of Geometry School and the Modern Mathematics Movement (MMM) through a comparative study between Brazil and France. This article represents the initial phase of this larger investigation, focusing on an analysis of how geometric transformations (GT) were incorporated into the Brazilian curriculum for students aged 11 to 14. The study focuses on secondary education standards between the 1950s and 1970s, with an emphasis on educational reforms and specific programmes in the state of São Paulo. The sources researched include: Simões Filho Reform (1951), Diário Oficial Programmes in 1965, and the Curriculum Guides from 1975. The analysis of documents shows appropriations of MMM principles within the Brazilian context, highlighting the role importance of GT as an integrative link between geometry and algebra. The analysis indicates that GT was meaningfully incorporated only in the 1975 Curriculum Guides, and even then, only in a suggestive, non-mandatory way, reflecting an adaptation of traditional geometric practices. The incorporation of GT also led to a reduced emphasis on deductive geometry in the latter years of secondary school (students aged 13 to 14), suggesting an attempt to modernize and diversify geometry teaching in the state of São Paulo.
Keywords: History of Mathematics Education. School Geometry. Modern Mathematics Movement. Secondary Education. Mathematics Programs.

RESUMO

A pesquisa apresentada faz parte de um projeto mais amplo que investiga as relações entre a História da Geometria Escolar e o Movimento da Matemática Moderna (MMM) por meio de um estudo comparativo entre Brasil e França. Para o presente artigo, como uma primeira etapa do projeto maior, iniciamos com um estudo que propõe analisar como as transformações geométricas (TG) foram integradas nas normativas curriculares do Brasil para alunos de 11 a 14 anos. O estudo foca nas normativas relativas ao Ensino Secundário entre as décadas de 1950 e 1970, com destaque para as reformas educacionais e programas específicos de São Paulo. As fontes examinadas foram: a Reforma Simões Filho (1951), os programas do Diário Oficial de 1965 e os Guias Curriculares de 1975. O exame dos documentos revela apropriações dos ideais do MMM no contexto brasileiro, destacando a importância das TG como elemento integrador entre geometria e álgebra. A análise dos resultados indica que as TG somente foram incorporadas de maneira significativa, em São Paulo, nos Guias Curriculares de 1975, de forma sugestiva, sem caráter obrigatório, refletindo uma adaptação das práticas tradicionais de geometria. A inclusão das TG exigiu a redução do enfoque na geometria dedutiva nas séries finais do 1º grau (alunos de 13 a 14 anos), indicando uma tentativa de modernização e diversificação do ensino de geometria no estado de São Paulo.
Palavras-chave: História da Educação Matemática. Geometria escolar. Matemática Moderna. Ensino Secundário. Programas de Matemática.

RÉSUMÉ

La recherche présentée fait partie d'un projet plus large qui examine les relations entre l'histoire de la géométrie scolaire et le mouvement des mathématiques modernes (MMM) à travers une étude comparative entre le Brésil et la France. Le présent article est la première étape de ce projet plus vaste, nous avons débuté par une étude visant à analyser la manière dont les transformations géométriques (TG) ont été intégrées dans les programmes scolaires brésiliens pour les élèves âgés de 11 à 14 ans. L'étude se concentre sur les normes relatives à l'enseignement secondaire entre les années 1950 et 1970, en mettant l'accent sur les réformes éducatives et les programmes spécifiques de l'état de São Paulo. Les sources examinées incluent la réforme Simões Filho (1951), les programmes du journal officiel de 1965 et les guides curriculaires de 1975. Ces documents révèlent les changements et les appropriations des idéaux du MMM dans le contexte brésilien, soulignant l'importance des TG comme élément intégrateur entre la géométrie et l'algèbre. L'analyse des résultats montre que les TG n'ont été incorporées de manière significative que dans les guides curriculaires de 1975, de manière suggestive, sans caractère obligatoire, reflétant une adaptation des pratiques traditionnelles de la géométrie. L'inclusion des TG a exigé une réduction de l'accent mis sur la géométrie déductive dans les dernières années du premier cycle de l'enseignement secondaire (élèves âgés de 13 à 14 ans), indiquant une tentative de modernisation et de diversification de l'enseignement de la géométrie dans l'État de São Paulo.
Mots-clés : Histoire de l'éducation mathématique. Géométrie scolaire. Mathématiques Modernes. Enseignement secondaire. Programmes de mathématiques.

INITIAL CONSIDERATIONS

The research presented here is part of a broader project that explores the relationship between the History of School Geometry and the Modern Mathematics Movement (MMM) through a comparative study between Brazil and France[4]. More specifically, the study initially aims to examine how geometric transformations (GTs) were integrated into the curricular standards of both countries for students aged 11 to 14. In a subsequent phase, the project will focus on the representation of GTs in key textbooks from Brazil and France.

A detailed examination of each country's curricular guidelines is essential as a first step in analyzing and understanding the role of GTs in geometry education during the MMM, especially in light of the cultural and educational distinctions between these nations. Between 1950 and 1970, Brazil experienced significant changes in educational regulation, transitioning from mandatory national guidelines to state-level frameworks, which were implemented more as curriculum suggestions than as strict requirements (as will be further discussed). By contrast, France maintained national, mandatory standards throughout this period, though specific programs saw more frequent, targeted adjustments.

The differences between the educational cultures of Brazil and France merit close examination to identify unique characteristics, allowing a comparative historical analysis to reveal how educational knowledge was appropriated and adapted through its circulation[5]. One primary source in this study is the secondary education curriculum developed by the *Organisation for European Economic Co-operation* (OEEC), published in France in 1961 and translated into Portuguese in 1962. Documents such as the Curriculum of 1961 are viewed as cultural agents (Burke, 2016), representing a synthesis of different cultural perspectives following extensive and productive debates. These documents serve as reference points for knowledge circulation, particularly between France and Brazil.

Thus, this comparative study between Brazil and France involves the complexity of, first, a collective discussing and producing a suggestion for a common curriculum and, second, the circulation of this program and its appropriation within the singularities of each country. Our primary focus is on this second phase: understanding how Brazil and France appropriated the shared curriculum proposal to develop their respective educational programs, with careful consideration of the following factors:

[4] Research project supported by FAPESP (São Paulo Research Foundation), Grant No. 2023/04639-8.

[5] Here and in what follows, we adopt Chartier's (1990, p. 26) concept of appropriation, according to which "appropriation, as we understand it, aims at a social history of representations, referring to the two fundamental determinations (which are social, institutional, cultural) and inscribed in the practices that produce them".

> The influence of dominant cultural models does not entirely preclude the possibility of their reception; rather, the imposition of new disciplines, the introduction of new forms of subjugation, and the establishment of new codes of conduct must invariably contend with and negotiate against deeply entrenched representations and shared traditions. (Chartier, 2009, p. 46-47, our translation).

The comparative approach adopted here facilitates an in-depth examination of how ideas are received and assimilated across different cultural contexts, enabling scholars to step outside their own familiar frameworks, to engage with diverse educational processes, and thus gain a deeper understanding of their own cultural assumptions (Valente, 2012).

Historically, research indicates a pronounced influence of French educational ideas on Brazilian education during the 19th century (Bastos, 2000). The MMM represented a broadly international movement, to which France made a notable contribution, particularly through the Bourbaki's work[6]. Brazil, for its part, welcomed French mathematicians and closely engaged with their theoretical foundations, although this was not an exclusively French influence, as we will explore in this analysis.

This article focuses specifically on Brazilian educational regulations governing Secondary Education – equivalent to what is now known as *Anos Finais do Ensino Fundamental* – over the period extending from the 1950s to the 1970s[7].

Regulatory structures for Secondary Education in Brazil were first established in the 1930s, as prior to this period, university admission could be achieved via a mixed system. This model included both sequenced instructions provided by secondary school and segmented examinations covering arithmetic, algebra, geometry, and trigonometry (Valente, 2004). The Francisco Campos Reform of 1931 structured Secondary Education into two distinct levels: the *Curso Fundamental* and the *Curso Complementar*, unifying multiple branches of study into a single subject designated as Mathematics[8]. The subsequent reform, known as the Capanema Reform of 1942, retained the two-cycle system but redefined the first cycle as a four-year *Ginásio* course, while the second cycle permitted either a scientific or classical orientate on, each with a three-year cycle (Bicudo, 1942).

In 1951, marking the outset of the period examined in this study, Ordinance No. 1,045, (December 14, 1951), enacted the Simões Filho Reform. This reform established a minimum

[6] Group Bourbaki refers to a group of French mathematicians (from the 1930s) who introduced an entirely new vision of their discipline. They conceived of mathematics based on the concept of structure and the axiomatic method. André Weil, Jean Dieudonné and Alexandre Grothendieck, French mathematicians from the Group, were in São Paulo (Brazil), and worked at the University of São Paulo (USP), respectively between 1945 and 1947, 1946 and 1948 and between 1953 and 1954.

[7] As part of our present project, a similar study is being prepared focusing on the French programs of that period.

[8] An in-depth analysis of the Francisco Campos Reform and the birth of mathematics in secondary schools can be read in *O Nascimento da Matemática do Ginásio* [The Birth of Gymnasium Mathematics] written by Valente (2004).

curriculum for Secondary Education while retaining the organizational framework of the Capanema Reform. Hence, the three aforementioned reforms (of 1931, 1942, and 1951) were each implemented nationwide. Beginning in the 1960s, however, a new legislative framework restructured Brazilian Secondary Education. The National Education Guidelines and Framework Law (LDB 4.024/61) empowered state education councils to define and develop subject-specific curricula, granting each state the autonomy to design its own educational program.

The state of São Paulo, chosen as the focal point of this study, was selected for analysis beginning in 1961, as it was a key locus for the diffusion of MMM ideals and served as a model for other states (Valente, 2008). Within this context, the São Paulo State Department of Education authorized two Mathematics programs for the *Ginásio* level (targeted at students aged 11 to 14). The first was published in the *Diário Oficial* of São Paulo on January 19, 1965, entitled "Suggestions for a Program Outline for the Mathematics Course" (GEEM, 1965a). Subsequently, Law No. 5,692, enacted on August 11, 1971, restructured Brazilian Basic Education, abolishing the separate primary and secondary courses, as well as the admissions examination, and instituting an eight-year Elementary Education cycle (*1º Grau*) for students aged 7 to 14, followed by a Secondary Education cycle (*2º Grau*) lasting three to four years for students completing the Elementary level. As a result, the second program analyzed in this study is titled "Curricular Guides Proposed for the Core Subjects of *1º Grau* Education" published by the São Paulo State Department of Education in 1975 (São Paulo, 1975).

The three documents analyzed here span the period from 1950 to 1970, with the Simões Filho Reform of 1951 serving as a national framework that shaped the Mathematics curriculum prior to the arrival of MMM in Brazil. The latter two documents, the program of 1965 published in the *Diário Oficial* and the Curricular Guides of 1975, regulated the Mathematics curriculum for the state of São Paulo, which took a pioneering role in adopting and promoting modernist educational principles in Brazil, as we will further demonstrate.

1. MODERN MATHEMATICS - INTERNATIONAL AND BRAZILIAN VIEW

The 1950s mark the early signs of a movement to reform mathematics education internationally, with significant developments taking place in various countries, particularly in Brazil and France. Notable milestones of this period include the establishment of the

Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) in 1950. This commission coordinated psychological, methodological, and practical efforts toward the improvement of mathematics education, involving professionals from diverse countries (Búrigo, 1989). Another landmark was the founding of the University of Illinois Committee on School Mathematics (UICSM) in 1951, which spearheaded secondary education reform by developing classroom materials, testing them in schools, and training teachers (D'Ambrosio, 1987). In addition, CIEAEM published a book featuring contributions from influential figures, including the epistemologist Jean Piaget and prominent French mathematicians Dieudonné, Choquet, and Lichnerowicz, along with the logician Evert W. Beth and educator Caleb Gattegno (Búrigo, 1989). The creation of the School Mathematics Study Group (SMSG) in 1958 further underscored these efforts, as mathematicians, mathematics teachers, educators, psychologists, and representatives of the scientific and technological communities came together with the goal of producing Modern Mathematics textbooks for secondary education (D'Ambrosio, 1987). According to Furinghetti and Menghini (2023, pp. 55-56),

> a reform movement spread around the world, usually known as "New Math" in most English-speaking countries or "modern mathematics"—the label that we will use—in most European countries. The difference in labels is not just a question of language. Modern mathematics had its initial inspiration in the Bourbakist theories, while New Math is usually identified with the movement which sprung out of the School Mathematics Study Group (SMSG) in the USA. The two labels cover very different phenomena in the various countries involved in the movement of reform […]

In Brazil, the early indications of proposals aimed at renewing mathematics education are evident in the National Congresses on Mathematics Education[9]. At the inaugural congress held in 1955 in Salvador (Bahia), there were no substantial references to the introduction of Modern Mathematics topics; however, significant discussions arose regarding the necessity to reorganize the Secondary Education curriculum (D'Ambrosio, 1987). In contrast, the second congress, which took place in 1957 in Porto Alegre (Rio Grande do Sul) marked the first notable advocacy for Modern Mathematics (Búrigo, 1989).

This period saw a proliferation of initiatives and mobilizations across both Europe and the Americas, particularly within the United States, concerning the discussion of proposals for the renewal of mathematics education. A pivotal event was the Royaumont Seminar, organized in 1959 by the OEEC, in France. The seminar culminated in recommendations for the reformulation of a curriculum intended to serve as a reference for various OEEC member

[9] The First Brazilian Congress on the Teaching of Mathematics took place in 1955 in Salvador/Bahia; the Second in 1957 in Porto Alegre/RS; the Third in 1959 in Rio de Janeiro/RJ; the Fourth in 1962 in Belém/PA; and the Fifth and final Congress in 1966 in São José dos Campos/SP.

states[10]. This new curriculum was collaboratively developed in a meeting in Yugoslavia in 1960 by a group of experts[11], resulting in the publication of *Un programme moderne de mathématiques pour l'enseignement secondaire* in Europe in 1961 (OECE, 1961). The curriculum was structured into two cycles of three years each, with the first cycle (for students aged 11-15) aimed at general education, while the second cycle was designed for those orienting towards advanced scientific and technical studies.

In parallel, despite interactions with European proposals and publications, Rios, Búrigo, and Oliveira Filho (2011) indicate that Brazil established connections with American initiatives for the modernization of mathematics education as early as 1946, notably through the establishment of the *Instituto Brasileiro de Educação, Ciência e Cultura* (IBECC). In 1960, IBECC São Paulo entered into an agreement with the Organization of American States (OAS) to facilitate Brazilian teachers' participation in continuing education programs at American universities. Among those who benefited from this opportunity was Osvaldo Sangiorgi[12], who undertook an internship at the University of Kansas from June to August 1960. Upon his return from the United States, Sangiorgi established the *Grupo de Estudos do Ensino da Matemática* (GEEM) in São Paulo in 1961[13]. The group was founded with objectives aligned with those of the SMSG and aimed to develop textbooks, organize congresses, translate educational materials, and conduct Modern Mathematics courses for educators (Rios, Búrigo & Oliveira Filho, 2011). Osvaldo Sangiorgi and his team at GEEM are regarded as instrumental in disseminating and facilitating the appropriation of the MMM in Brazil, particularly within the state of São Paulo.

> Understanding the trajectory of the Modern Mathematics Movement (MMM) in Brazil necessitates a thorough examination of the contributions of Osvaldo Sangiorgi. Sangiorgi engaged in a multifaceted approach, articulating proposals and programs while uniting a diverse array of professionals to establish the *Grupo de Estudos do Ensino da Matemática* (GEEM). He maintained strategic connections with governmental bodies and various media outlets, including print and television. Additionally, he authored a widely acclaimed series of textbooks that became essential references for the MMM at the secondary education level, achieving significant editorial success. Moreover, Sangiorgi was an active participant in both national and international conferences and meetings that addressed the implementation and advancement of the MMM. (Matos & Leme da Silva, 2011, p. 180, our translation).

[10] The member countries were: Austria, Belgium, Denmark, France, Germany, Greece, Ireland, Iceland, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom (GEEM, 1965b).

[11] The experts who drew up the program were, according to OECE (1961): Emil Artin (Germany), Otto Botsch (Germany), Gustave Choquet (France), Bozidar Derasimovic (Yugoslavia), Howard Fehr (USA), Cyril Hope (England), Erik Kristensen (Denmark), Djuro Kurepa (Yugoslavia), Paul Libois (Belgium), Laurent Pauli (Switzerland), Lennart Rade (Sweden), Bruno Schoeneberg (Germany), Willy Servais (Belgium), Marshall Stone (USA), Pierre Theron (France) and Mario Villa (Italy). (GEEM, 1965b). Botsch was one of the mathematicians who propose an approach to geometry based on the notion of transformation (Calame, 1979, p. 15).

[12] Osvaldo Sangiorgi (1921–2017) earned his degree in Mathematical Sciences in 1941 from the University of São Paulo (USP). He was a renowned mathematics teacher and a prolific author of textbooks starting from the 1950s. A comprehensive study on Professor Osvaldo Sangiorgi can be found in *Osvaldo Sangiorgi: um professor moderno* (Valente, 2008).

[13] A comprehensive study on GEEM can be found in *Osvaldo Sangiorgi: um professor moderno* (Valente, 2008).

The book *Un programme moderne de mathématiques pour l'enseignement secondaire*, published in France in 1961, was translated into Portuguese by Professor Jacy Monteiro[14] and published by Sangiorgi's Group (GEEM, 1965b)[15]. The program published in the *Diário Oficial* of the State of São Paulo in 1965 was also disseminated by the GEEM under the title *Matemática Moderna para o Ensino Secundário* (GEEM, 1965a).

The next program to be examined, published a decade later, was issued by the Secretary of Education of the State of São Paulo in 1975, in a document titled *Guias Curriculares Propostos para as Matérias do Núcleo Comum do Ensino de 1º Grau*. The mathematics team was comprised of Professors Almerindo Marques Bastos, Anna Franchi, and Lydia Condé Lamparelli. This document reflects a very different educational moment and context compared to the 1960s, characterized by a wave of critiques and negative evaluations of the MMM. By this time, national congresses on mathematics education had ceased in Brazil, and in São Paulo, the GEEM conducted its final teacher training course for entry into the teaching profession in 1976 (Leme da Silva, 2006).

As previously mentioned, this study focuses on a specific component of geometry: geometric transformations (GTs), which Felix Klein (1849-1925) emphasized for secondary education in 1872 when he asserted that "the concept of transformation plays a vast coordinating and simplifying role in the study of geometry" (Catunda et al., 1990, p. 13). In Brazil, the Campos Reform of 1931 incorporated modernizing ideals established by the International Commission on Mathematical Instruction (ICMI)[16], which Klein chaired in the early 20th century, by introducing fundamental notions of variable quantities and functional dependence into mathematical concepts (Schubring, 2003). Additionally, the method of GTs involves applying the properties of functions within geometry, an approach similarly advocated by Klein (Alves & Galvão, 1996).

After 1931, the topic of GTs gradually diminished in significance in subsequent reforms, culminating in 1951 where it became a marginal topic within geometry education, limited to a brief presentation of translation and rotation, without a clear pedagogical objective (Jahn & Guimarães, 2023). During the discussions surrounding the MMM, GTs re-emerged as a key focus[17] and were integrated into the *Modern Mathematics Program*, developed in Yugoslavia

[14] Professor Luiz Henrique Jacy Monteiro (1921-1975), a mathematician who graduated from the University of São Paulo (USP) in 1943, was Jean Dieudonné's assistant when he was at USP in 1946 (Duarte, 2007).
[15] The edition we consulted dating back to 1965.
[16] Known in French as the *Commission Internationale de l'Enseignement Mathématique* (CIEM).
[17] In Brazil, a study at the end of the 20th century considered GTs to be one of the causes of the supposed abandonment of geometry teaching in the MMM (Pavanello, 1993). Later, a historical review called into question the role of GTs in the MMM

in 1961, as a means to link geometry with algebra. This document stated that the two subjects "should initially be studied from a physical and intuitive standpoint to explore the properties of figures through methods such as folding, reflection, rotation, translation, cutting, and using regularly spaced points on a circumference and regular polygons" (GEEM, 1965b, p. 70).

From the outset of the section titled "Functions and Applications," it is noted that geometry employs numerous functions that correspond to figures regarded as sets of points; these are referred to as "geometric functions" to distinguish them from numerical functions. In this context, the concept of transformation appears as bijective functions (GEEM, 1965b, p. 39), highlighting symmetry with respect to a line or point, translation, rotation, and homothety as elementary transformations in geometry, along with the study of compositions of these transformations. Furthermore, one objective of the geometry course is to "seek properties that remain invariant under physical and algebraic transformations" (GEEM, 1965b, p. 69). Thus, albeit intuitively and with an emphasis on experimentation, it can be asserted that a functional approach to GTs is outlined in this proposal.

The broader project mentioned in the introduction aims to analyze the integration of this content into teaching and its role in the development of school geometry during the MMM period in both Brazil and France. We began with a separate analysis of the regulations, first in the Brazilian context, and in a subsequent article, in the French context. In the third stage, through a comparative study, we will examine the textbooks. In this article, we will focus on the Brazilian regulations from 1951 and the programs from 1965 and 1975 in the State of São Paulo, which we will now analyze.

2. GTs AND THE SIMÕES FILHO REFORM FROM 1951

The examination of the presence and objectives of GTs within the Simões Filho Reform in 1951 has been explored by Jahn and Guimarães (2023), whose analysis underpins this study. In the Mathematics program from 1951, titled "Minimum Program for Secondary Education," the concepts associated with geometry were confined to the 3rd and 4th grades of the G*inásio* level. Specifically, only two GTs were outlined for the 3rd grade: translation, introduced after

pointed out by the aforementioned study, using other sources and historiographical references (Leme da Silva, 2022). In any case, the study of GTs and the MMM deserves to be explored further.

the exploration of the properties of parallelograms and trapezoids, and rotation, which concluded the study of circles and circumferences.

The concept of function, initially introduced in the Reform from 1931 but omitted in 1942, remains absent from the Reform of 1951 in the curriculum for the *Ginásio* (1st Cycle). This absence suggests that the approach to GTs was framed in terms of the properties of geometric figures rather than through a functional lens. The methodological guidelines for Mathematics indicate that in the 1st Cycle, during the "initial years of the *Ginásio*," instruction should be predominantly practical and intuitive. The aim is to gradually foster within students a recognition of the need for justification, proof, and demonstration, while also introducing the deductive method, albeit with the requisite caution (Brasil, 1952, p. 240).

From these guidelines, we can infer that while highlighting the significance of intuitive studies as a foundational step, geometry limited to the final two grades must also incorporate deductive reasoning, with care taken to avoid lengthy and cumbersome proofs. Regarding GTs, we echo the conclusions drawn by Jahn and Guimarães (2023, p. 347) concerning the reforms made in 1942 and 1951: "In these last two reforms, the role of GTs in geometric education is neither clearly defined nor justified, resulting in content that is practically 'isolated' and not even included in the exercises found in the reviewed textbooks." We will now proceed to the subsequent period, corresponding to the 1960s.

3. GTs AND THE 1965 MATHEMATICS PROGRAM OF SÃO PAULO

As previously discussed, the 1960s introduced substantial changes to the organizational structure of Brazilian educational policy, notably granting states greater autonomy in developing their own curricula. This period also brought a reclassification of educational cycles: the 1st Cycle was renamed *Ginásio*, now comprising four years, and the 2nd Cycle became known as *Colégio*, with a duration of three years.

Regarding curricular development, the GEEM, established in 1961, played a critical role in discussions and proposals for a new mathematics curriculum, integrating contemporary and internationally circulating reformist ideas, which had particularly gained traction in Brazil. Among GEEM's various initiatives, the proposal presented at the *IV Congresso Brasileiro do Ensino da Matemática* (CBEM), held in Belém (Pará) in 1962, stands out. This document, titled *Assuntos mínimos para um programa de Matemática Moderna para o Secundário* ("Minimum

Topics for a Modern Mathematics Program for Secondary Education”), represents one of the earliest formal efforts to incorporate Modern Mathematics principles into Brazil's mathematics curriculum.

It is important to note that GEEM’s membership was diverse, comprising a broad range of professionals, including professors with doctorates in mathematics from the University of São Paulo (USP), secondary school mathematics educators, educational researchers, and other advocates for modernizing mathematics education. Led by Sangiorgi, GEEM submitted the *Minimum Topics for a Modern Mathematics Program* to the participants of the IV CBEM, where the proposal received “unanimous approval” (GEEM, 1965a, p. 89). The introduction to this document underscores the following guiding principles:

> The primary aim of Modern Mathematics Programs (which is the recommended terminology) is to explore *the same fundamental subjects of Mathematics* deemed essential for the education of young students. However, this is to be accomplished using a *modern language* that resonates more with the new generations. This modern approach fundamentally incorporates *the concept of sets* and is designed to facilitate *the development of mathematical structures*. Such structures enable students to leverage their existing mental frameworks more effectively and with less effort, while also highlighting the distinctive nature of Mathematics as it is understood today. (GEEM, 1965a, p. 89, our emphasis).

This passage encapsulates the essence of the introduction of Modern Mathematics into Secondary Education, underscoring continuity in the exploration of fundamental mathematical concepts but approached through a revitalized framework. Rather than a complete curricular overhaul, the aim was to deliver these concepts in a manner that is more accessible and engaging for students, employing a contemporary language rooted in set theory, which supports the development of mathematical structures.

With regard to geometry in the curriculum for the *Ginásio*, the foundational elements are presented under the section “Fundamental elements of plane geometry: point, line, ray, segment, plane, half-plane, angles, bisectors.” This point in the curriculum is explicitly identified as the opportunity to introduce deductive geometry. Notably, there is a recommendation to privilege intuitive understanding in the presentation of geometric concepts prior to moving into deductive processes. This approach aligns closely with that of the Simões Filho Reform (1951), as analyzed in the previous section. Moreover, the use of set theory language to define and characterize geometric objects is emphasized, representing a distinctly modernizing aspect.

According to the “Minimum Topics” outlined in 1962, the study of deductive geometry remains concentrated in the final two years of the *Ginásio* (targeting students aged 13-14), much like the framework proposed in the Reform from 1951. A key change, however, is the reintroduction of the concept of function in the second year of the *Ginásio*. Despite this, GTs are absent from the curriculum for the *Ginásio*, and the two transformations included in the previous reform were not retained.

Three years later, in 1965, the state of São Paulo published a document titled "Suggestions for a Program Outline for the Mathematics Subject" in the *Diário Oficial*. This document was developed by the Secretariat of the State of São Paulo via a commission led by professors Benedito Castrucci[18] (chair) and Osvaldo Sangiorgi (secretary), both prominent figures within GEEM. Both proposals, in the 1962 and 1965, reflect significant influence from GEEM's deliberations. When comparing these two documents, we note the reintroduction of GTs in the curriculum for the third-year *Ginásio*, as displayed in Table 1 below.

Table 1 – Geometry and Transformations in the Proposals for *Curso Ginasial* in the 1960s

Curriculum	Organization of the Geometry	Instructions/Orientations	Contents for GTs
Minimum Topics GEEM (1962)	Plane Euclidean Geometry 3rd and 4th grade	- Introduce the fundamental concepts of plane geometry intuitively. - Utilize the language of sets and their operations whenever possible. - Illustrate how some properties emerge as consequences of more basic principles, highlighting the deductive process inherent in Geometry.	Not included
Suggestions for a Program Outline for the Mathematics Subject (1965)	1st, 3rd and 4th grade	- Intuitive study of the key geometric figures (1st year of middle school) - Introduction to Deductive Geometry in the 3rd and 4th years of middle school.	Transformations in the 3rd grade: Translation, rotation and symmetry

Source: Prepared by the authors based on GEEM (1965a).

In reviewing Table 1, two significant shifts emerge between the proposals from 1962 and 1965. First, we observe the addition of intuitive geometry in the first year, preceding the deductive approach that continues in the third and fourth years. The second notable change is the reintroduction of GTs in the curriculum for the *Ginásio*, specifically in the third year of the 1965 program, where translation, rotation, and symmetry are now included. Meanwhile, functions appear in the fourth year, though without a direct link to GTs.

To grasp the reasoning behind the inclusion of GTs in 1965 but not in 1962, it's essential to consider the authors behind each proposal. Evidence suggests that the inclusion of GTs in the curriculum for the *Ginásio* was a topic of active discussion within GEEM, especially among

[18] Benedito Castrucci (1921–1975), Professor of Geometry and faculty member at USP, was responsible for the Geometry courses at GEEM.

professors Benedito Castrucci and Osvaldo Sangiorgi, who authored the first modern textbooks published in São Paulo. Sangiorgi's book, published in 1967, and the textbook by Alcides Bóscolo and Castrucci, published in 1970, were both written by these professors, who also led the commission behind the program of 1965, issued by São Paulo's Department of Education. In both textbooks, the authors placed GTs in an appendix at the end of the third-year content, rather than in the main body, suggesting a nuanced approach to these topics[19]. This choice may reflect a dual motivation: adherence to the curriculum of 1965 recommendations on one hand, and, on the other, the authors' own stance on integrating GTs into the curriculum for the *Ginásio*. This positioning suggests a possible divergence, as evidenced by the absence of GTs in the proposal made in 1962.

We now proceed to examine the final curriculum, which incorporates GTs and presents objectives, recommendations, and teaching strategies for effectively integrating them.

4. GTs AND THE SÃO PAULO CURRICULUM GUIDE (1975)

The São Paulo *Curriculum Guide* [Guia Curricular], as previously noted, introduces a range of significant innovations within the educational framework. This document represents the first regulatory framework implemented in São Paulo following the integration of primary and secondary education into an eight-year of *ensino de 1º grau*. Throughout the 1970s, Modern Mathematics acquired new significance both on the international stage and within Brazil. The release of Morris Kline's *Why Johnny Can't Add: The Failure of the New Math* in 1973, along with its subsequent Portuguese translation, *O Fracasso da Matemática Moderna*, in 1976, underscores the rise of critical discourse surrounding MM (Kline, 1973, 1976)[20].

In 1980, the newspaper *Folha de São Paulo* published an article titled *Denunciada na USP falência da Matemática Moderna*, featuring an interview with Professor Elza Furtado Gomide from the Department of Pure Mathematics at the University of São Paulo (USP). During this interview, Professor Gomide discussed a study she conducted alongside other faculty members regarding the adverse effects stemming from the implementation of MM (Nakashima, 2007).

[19] A comprehensive study on geometric transformations in the textbooks by Sangiorgi and Bóscolo & Castrucci can be found in Leme da Silva and Jahn (2024).

[20] Morris Kline (1908-1992) was a mathematician and historian of mathematics, as well as a professor of mathematics at the Courant Institute of Mathematical Sciences, New York University.

Additionally, the team tasked with developing the mathematics proposal for the "Curricular Guidelines for Core Subjects of Education in *1º. Grau*" in early 1972 comprised educators from the São Paulo state school system and the IBECC. This shift indicates a departure from the leadership previously held by the GEEM in formulating these guidelines. The three authors credited for the mathematics section of the Guidelines – Almerindo Marques Bastos, Anna Franchi, and Lydia Condé Lamparelli – were all educated at USP. In October 1972, a critical assessment of the initial version of the program was conducted, receiving commendations from Ubiratan D'Ambrosio[21] while also facing critiques from authors of mathematics textbooks (Lamparelli, 2018). Nevertheless, the final version was not published until 1975.

In presenting the proposal for the Guidelines, one can identify substantial alterations compared to earlier initiatives. Notably, one of the program's overarching objectives is to "recognize the interrelationship among various fields of mathematics" (São Paulo, 1975, p. 212). The text of the program emphasizes that "the concepts of relation and function must always be highlighted in all situations" (São Paulo, 1975, p. 214), given that the program of 1965 had no guidelines, only a list of contents distributed by grade. The general guidelines for teaching geometry also reflect these directives.

The eight-year program stipulates that the subjet of applications and functions should be incorporated across all grades, with the goal of acquiring a language and conceptual framework that serve as unifying elements of mathematics while in the program of 1965, function is only included in the last grade of *Ginásio*. In the initial four years and the seventh year, these themes should be addressed implicitly through various activities and problem-solving exercises, whereas in the fifth, sixth, and eighth grades, they should be explicitly integrated. Indeed, one of the program's primary objectives is to "recognize the interrelationship among various fields of mathematics" (São Paulo, 1975, p. 212). The text reiterates that the concepts of relation and function "must always be highlighted in all situations" (São Paulo, 1975, p. 214), with the general guidelines for teaching geometry mirroring these directives.

> In the first four years, Geometry should be approached as an exploration of the observable physical space, starting with intuitive topological concepts such as interior, exterior, and boundary. This should be followed by the identification of common geometric shapes found in the physical world. Knowledge should be cultivated through observation and manipulation of appropriate teaching materials. *Even in the following four years, the emphasis on intuition should remain, rooted in experiential learning and observation*. Concepts from Set Theory should be integrated as supportive tools, while problem-solving should incorporate methods

[21] Ubiratan D'Ambrosio (1933–2021) was a mathematician, historian, and philosopher of mathematics, best known for developing the Ethnomathematics Program. He taught at several Brazilian institutions (Unicamp, USP, UNESP Rio Claro, PUC/SP, PUCAMP, UNIAN) as well as in the United States at Brown University and New York University.

> beyond purely geometric ones. Results obtained intuitively should serve as a foundation for deriving additional properties through straightforward and concise deductions. *The concept of transformation should be highlighted whenever possible, with a focus on exploring invariant properties under such transformations*. Furthermore, the idea of oriented segments should be introduced, laying the groundwork for the subsequent understanding of vectors. The concept of area may be presented using graph paper, where students can count the squares contained within a figure. (São Paulo, 1975, p. 218, our emphasis)

Two key issues warrant a comprehensive analysis in the context of educational reforms in geometry. First, the emphasis on intuitive geometry is consistently reinforced throughout all academic years, facilitated by hands-on experiences and observational learning, extending even beyond the fifth grade. At the beginning of its presentation, the *Guide* explicitly advises against an axiomatic approach for primary education, advocating instead for an intuitive framework wherein mathematical concepts are developed through student engagement and the manipulation of tangible materials. In our view, this gradual progression from concrete to abstract thinking is particularly crucial for teaching transformations, as it allows students to grasp these complex ideas in a more accessible and practical manner. In contrast to the educational reforms from 1951 and 1965, the focus on mathematical deduction has significantly diminished.

The second critical aspect concerns the study of transformations itself, which is explicitly addressed from a functional perspective that emphasizes invariant properties. Below, we provide a detailed analysis of how transformations have been introduced and discussed, as illustrated in Table 2.

Table 2 – GTs in the São Paulo *Curriculum Guide* (1975)

Grade	Content of GTs	Main Objective	Approach to GTs
6th *Intuitive geometry and geometric constructions*	Notion of transformation	- Relate the concept of a function to that of a transformation in the Euclidean plane. - Recognize isometry as a transformation that preserves distances. - Identify congruent figures as those that can be mapped onto each other via an isometric transformation.	Functional Transformations as Functions**:** In geometry, transformations can be understood as functions; this includes the study of their invariant properties and the notion of congruence based on the concept of isometry.
	Concept of Parallelogram. Notion of Translation	- Determine the image of a segment corresponding to another segment through translation. - Associate the concept of a parallelogram with the properties of translation.	
7th *Beginning of the use of hypothetical-deductive reasoning in geometry*	Axial and Central symmetry	- Understand axial and central symmetry as transformations of the plane. - Construct the symmetric images of given points with respect to a line or a point. - Identify the axes or centers of symmetry in a geometric figure. - Determine the symmetric figure of a given figure concerning a specific axis or center of symmetry. - Identify the invariants under axial or central symmetry.	Functional Axial and Central Symmetry as Transformations of the Plane (within itself): This concept is associated with the study of figures, including determining symmetric figures and identifying their axes or centers. Translation as a Transformation Associated with the Study of Parallelograms.
	Translations	-Construct corresponding points through translation. - Identify the invariants under translation.	
8th *Homothety and similarity: Applications*	Homothety	- Develop a more comprehensive understanding of the concept of transformation. - Determine the homothetic image of a given point. - Relate the value of the ratio to: • the position of homothetic points relative to the center of homothety; • the enlargement, preservation, or reduction of the geometric figure. - Identify the invariants under homothety.	Functional Homothety as a GT associated with the study of similar figures.

Source: Prepared by the authors based on São Paulo (1975)

The analysis of Table 2 underscores the key elements previously highlighted in the recommendations for teaching geometry outlined in the *Curriculum Guide* from 1975. GTs are thoughtfully distributed over three academic years, beginning with the introduction of the concept of transformation and establishing a clear link between the notions of function and transformation within the geometric plane. Isometries are characterized as transformations that maintain distance, and the specific transformations covered include axial symmetry, central symmetry, translation, and homothety.

When examining elements that illustrate the connections between these transformations and the principles of classical Euclidean geometry, it is essential to emphasize the relationships between congruence and isometry, the connection of translation with the study of parallelograms, and the association of homothety with similar figures. These relationships help to contextualize the purpose and objectives behind the introduction of this content at the specified educational level. The study of GTs can be seen as a practical application of the concepts of function and congruence, laying down vital foundations for grasping the principles of classical Euclidean geometry.

Viewed from this lens, the proposal from 1975 reveals a distinct approach compared to earlier frameworks, while still honoring the core principles of MM. These principles encompass the integration of topological concepts in the early grades, the utilization of set theory as an auxiliary tool, and the study of transformations. In a context marked by criticisms of MM, this proposal can be interpreted as a second wave of appropriation of the movement within the state of São Paulo, prompting a deeper examination of the trajectories and experiences of its authors.

Almerindo Marques Bastos, one of the key authors, reflects on his experience as a student of Jacy Monteiro, the professor responsible for translating the Modern Program developed by the OEEC in 1961. He was recruited by the *Divisão de Assistência Pedagógica* [Division of Pedagogical Assistance] (DAP) and received funding from the Ministry of Education and Culture, alongside support from the DAP and the *Centro de Recursos Humanos e Pesquisas Educacionais* (CERHUPE). Almerindo emphasizes that his involvement in drafting the Guide was contingent upon its non-mandatory nature, as he, being a teacher in the system, was reluctant to be subject to such impositions. He underscores the term "proposed" in the title of the *Guias Curriculares Propostos para as matérias do Núcleo Comum do Ensino de 1º grau* [Proposed Curriculum Guides for Core Subjects of Education in *1º. Grau*"] (Souza, 2005).

Educator Ana Franchi, who after completing her studies at the *Escola Normal* worked as a primary school teacher since the 1960s, later enrolled in the Mathematics Teaching degree program (*Licenciatura*) at USP. Likewise, Lydia Lamparelli worked at the IBECC in collaboration with the Secretariat of the State of São Paulo. She translated SMSG textbooks into Portuguese with Lafayette de Moraes and adapted them to fit the Brazilian context in the late 1960s. Lamparelli corroborates Almerindo's account, noting that the *Curriculum Guide* was often misconstrued as an official program and that its text was intended for teachers rather than students (Lamparelli, 2018).

In essence, the individuals involved in developing the proposal of 1975 present distinct profiles compared to the authors of 1965. The context of the 1970s reflects a period of reappraisal of modernizing ideas, highlighting a refinement of concepts previously proposed.

A close reading of the Guide from 1975, taking into account the emphasized points, reveals notable similarities with *Un programme moderne de mathématiques pour l'enseignement secondaire* developed by the OECE in 1961, which was translated and published by the GEEM in 1965. These similarities primarily relate to a more intuitive approach to geometry and an emphasis on the concept of function, with transformations emerging as specific functions within geometry. For instance, examples of initial exercises involving transformations proposed in the Modern Program (GEEM, 1965b) pertain to the topics "Functions and Applications" and "Composition of Two Functions," which suggest, respectively, providing examples of functions in geometry and composing two isometries (GEEM, 1965b, pp. 30 and 41). The notion of linking congruence to isometry is also evident in the Modern Program (GEEM, 1965b, p. 43), tied to an algebraic approach to a relation of equivalence.

In the specific section dedicated to the program of geometry (GEEM, 1965b, p. 68), there is a recurring emphasis on the use of "material models (facilitating observation and experience)" as "the foundation from which mathematical abstraction can be cultivated." From this viewpoint, it is recommended that transformations be introduced in a tangible manner from the outset, as they "retain a kinematic aspect that contrasts with the static nature of mathematics and enables students to transition seamlessly between the two" (GEEM, 1965b, p. 68).

The stated objectives for the geometry course further validate our interpretations and highlight the role of transformations at this educational level, aligning once again with what is suggested in the Curriculum Guides from 1975, particularly the aim of "seeking invariant properties under physical and algebraic transformations" (GEEM, 1965b, p. 69).

In conclusion, both regulatory documents – the *Modern Program* of the OECE published in Brazil in 1965 and the *Curriculum Guide* from 1975 – emphasize the significance of the unity of mathematics, the structure of algebraic systems, and the fundamental concept of function, which is essential for the study of both numerical functions and geometric transformations.

CONCLUDING REMARKS

This study revisited three pivotal historical periods – the 1950s, 1960s, and 1970s – and analyzed three distinct reforms or curricular proposals: the Simões Filho reform, the Suggestions for a Program Outline from the São Paulo State Education Department (SEE-SP), and the Curriculum Guides for mathematics education in the first cycle of Secondary Education (known as G*inásio* until 1971, subsequently referred to as Elementary Education).

In our analysis, it is essential to emphasize the distinctions among these periods and reforms. In the first reform, which was national in scope and official in nature, we had no access to traces that would allow us to understand its backstage, particularly regarding the team responsible for developing the mathematics curriculum. In contrast, the latter two reforms are state-level initiatives from São Paulo, presenting suggestions and prescribed curricula that questioned the official and mandatory character. Moreover, it is vital to consider the Brazilian context, which is characterized by significant economic, social, cultural, and educational disparities. Our work, therefore, represents an effort to understand the dynamics of incorporating and resisting new knowledge within an entrenched school culture, especially during a politically charged era marked by substantial expansion of Secondary Education in São Paulo. Valente (2008) notes the level of socioeconomic growth in São Paulo during the 1950s, which led to extensive urbanization. In the education sector, secondary school enrollments nearly doubled over the decade, reaching approximately 360,000 students by 1960.

Consequently, our reflections do not aim to generalize the findings to the entire Brazilian context, given the focus on the state of São Paulo and the exclusive analysis of regulatory frameworks, which may not necessarily mirror actual classroom practices. Nevertheless, we can assert that these frameworks indicate guidelines and power dynamics that should not be overlooked in a more nuanced analysis, particularly when compared to the proposals articulated in textbooks, as outlined in the continuation of this project.

Having made these observations, we have synthesized the various proposals into a summary table, underscoring the presence of transformations and their relationships (or lack thereof) with functional approaches. Since the 19th century, the integration of functions with geometry, algebra with geometry, and movement with geometry has been a subject of ongoing discussion, encompassing the exploration of more intuitive methods, as depicted in Table 3.

Table 3 – Summary of GTs in the Curricular Proposals of 1951, 1965, and 1975

Reform	GTs	Concept of function	Geometry approach
Simões Filho reform (1951)	Translation and Rotation in the 3rd grade	Not present	Deductive Geometry in the 3rd and 4th grades
Suggestions for a Program Outline SEE-SP (1965)	Translation, Rotation and Symmetry in the 3rd grade	Present in the 4th grades	Deductive Geometry in the 3rd and 4th grades
Proposed Curriculum Guide for Mathematics (1975)	Plane Transformations, Isometry, Translation, Axial and Central Symmetry and Homothety, from the 6th to the 8th grade	Present throughout all eight years, explicitly included in the 5th, 6th, and 8th grades	Intuitive Geometry throughout all eight years, with a reduction in Deductive Geometry

Source: Prepared by the authors

The analysis of Table 3 reveals that the incorporation of transformations within the set of regulations examined is explicitly present and purposefully defined only in the Curriculum Guides from 1975, which coincides with a period of decline for the MMM. It is crucial to note, however, that these transformations are framed as suggestions rather than requirements, contrasting sharply with the mandates established by the reform from 1951.

This suggests that it may have been necessary to "soften" the official stance to facilitate the integration of the study of transformations into the proposals, presenting them as options rather than a singular, compulsory approach. All indications point to the fact that this area of knowledge – transformations – necessitates substantial changes to what has traditionally been considered Euclidean Geometry, which has held undeniable importance within the regulations. Moreover, for transformations to gain prominence in the latter years of elementary education as outlined in the *Curriculum Guide* from 1975, it was necessary to reduce the emphasis on and significance of deductive geometry in the final two grades of the course.

In conclusion, we underscore the distinctiveness and potential of transformations in the ongoing effort to establish an integrated and cohesive mathematics curriculum, a goal that has consistently been pursued despite the many challenges that have made full realization difficult. Future studies will extend the examination of transformations regulations in the French context during the same period. Subsequently, a comparative analysis of textbooks from both countries will be conducted to deepen our understanding of the MMM and the teaching of geometry.

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