Revista de História da Educação Matemática
HISTEMAT

ISSN: 2447-6447 **Submetido:** 03/04/2025 **Aprovado**: 14/08/2025

https://doi.org/10.62246/HISTEMAT.2447-6447.2025.11.758

Ensinar matemática por meio de atividades históricas: duas experiências pedagógicas

Teaching Mathematics Through Historically-Based Activities: two Educational Experiments

Marc Moyon[1]
IREM de Limoges, Université de Limoges
marc.moyon@unilim.fr
Orcid: https://orcid.org/0000-0002-5773-5513

Frédérique Plantevin[2]
IREM de Brest, Université de Bretagne Occidentale
frederique.plantevin@univ-brest.fr
Orcid: https://orcid.org/0000-0002-0079-6702

[1] Docteur en histoire et philosophie des mathématiques de l'Université de Lille (France). Professeur d'histoire des mathématiques de l'université de Limoges, CNRS, XLIM, UMR 7252, F-87000 Limoges, France. E-mail: marc.moyon@unilim.fr.

[2] Docteur en physique mathématique de l'Université d'Aix-Marseille (France). Maître de conférence en mathématiques de l'université de Brest, CNRS-LMBA UMR 6205, France. E-mail: frederique.plantevin@univ-brest.fr.

RESUMO

Este artigo apresenta dois exemplos de atividades baseadas na história para o ensino de matemática a alunos de 8 a 11 anos. Elas diferem em vários aspectos: seus recursos históricos são, respectivamente, uma prancha de desenho do *Códice Atlântico*, de Leonardo da Vinci, e uma máquina de calcular com rodas, datada da década de 1950 — períodos distintos, objetos bastante diferentes e conteúdos matemáticos diversos. No entanto, também apresentam semelhanças: em ambas, utilizam-se artefatos interdisciplinares, cujo potencial pedagógico é explorado e apresentado de forma intencionalmente articulada. Essas duas experiências integram um projeto mais amplo, que busca produzir materiais concretos e precisos para professores que atuam com essa faixa etária e desejam introduzir uma perspectiva histórica em suas aulas. Esse projeto insere-se em uma longa tradição de estudos sobre a história e a epistemologia da matemática em contextos educacionais, desenvolvidos no âmbito da rede IREM (*Institut de Recherche sur l'Enseignement des Mathématiques*), na França. Esse contexto, atualmente inserido em um campo internacional de pesquisa, é apresentado no início do artigo. Em seguida, as duas atividades são descritas em detalhes, com seus objetivos a priori, análises das produções dos alunos e materiais completos para o leitor, seguindo a abordagem proposta pelos autores..

Palavras-chave: História da matemática; geometria plana; numeração; cálculo; interdisciplinaridade.

ABSTRACT

This article presents two examples of history-based activities in mathematics education for students aged 8 to 11. They are different in many ways; their historical resources are respectively a drawing board from Da Vinci's Codex Atlanticus and a wheeled calculating machine from the 1950s: different periods, very different objects, different mathematics approached; but they are also similar: for both, interdisciplinary artifacts are used, pedagogical potential is explored and rendered in a deliberately coordinated way. The fact is, these two experiments are part of a wider project to produce precise, concrete content for teachers (of these 8-11 year-old students) wishing to introduce a historical perspective into their teaching; this wider project is itself in some ways the result of a fairly long tradition of studying the history and epistemology of mathematics in an educational context in the IREM (*Institut de Recherche sur l'Enseignement des Mathématiques*) network in France. This context, also situated in this now international field of research, is explained, then the two activities are described in detail, with a priori objectives and analyses of the students' productions, a complete material for the reader presented according to the approach shared by the authors.

Keywords/Palabras clave: History of mathematics; plane geometry; numeration; calculation; interdisciplinarity

INTRODUCTION

In 2018 was published the first edition of a quite unique book: nine chapters covering the French curriculum of mathematics for children between 8 and 11 years old, through historically-based activities, all of them original and tested in classrooms. It was the result of the project “Passerelles” involving nine research groups in the national IREM (Institute for Research in the Teaching of Mathematics) network within the commission inter-IREM “Epistemology and history of mathematics”[3]. Each local group was constituted of teachers of primary and secondary[4] schools, and one – at least – university teacher, in the usual way of IREMs, additionally with a national coordination. The resulting book (Moyon & Tournès, 2018) was a collective work of which the authors of this article took part as group leaders in their respective IREM, the first author being moreover one of the two directors of the whole book.

The commission inter-IREM “Epistemology and history of mathematics” was the cradle of introducing historically-based activities in mathematics teaching in France and a pioneer abroad; since its beginning (1975), this national group has produced a large number of collective books exploring this idea (the list of them can be seen through the link in note 1), most of them in French, of course. There is a noticeable exception with *Let history into the mathematics classroom* (Barbin *et al.*, 2017), whose aim was to make the work carried out in the IREM network visible and accessible to non-French speakers.

Actually, there are many empirical research about interactions between mathematics and its history in the school context around the world, just as well more generally, between sciences and their history, in education; numerous international commissions aim to promote this research at periodic conferences (ESU and HPM for example). The literature on the subject is extremely abundant; the bibliographic synthesis (Clark *et al.*, 2016) or the two works (Fauvel & Van Maanen, 2000) and (Katz and Tzanakis, 2011) which provide a detailed overview of classroom practices give an good idea of that. This research continues to develop as evidenced by recent publications (Moyon, 2024) or the recent and special issues of the international journal *ZDM-Mathematics Education* untitled “History of mathematics in mathematics education: Recent developments in the field”[5] (54, 2022) and the French journal *Petit x*[6] on History of Mathematics and Didactics (122, 2025)[7]. Regarding History of Science

[3] http://www.univ-irem.fr/spip.php?rubrique15
[4] Since in France, students are in primary schools till their 10th year and then go to secondary schools although they are considered as part of a unique learning cycle (the third one).
[5] https://link.springer.com/article/10.1007/s11858-022-01442-7
[6] https://irem.univ-grenoble-alpes.fr/revues/petit-x/
[7] One of the authors of this publication has written two articles on mathematics textbooks and the history of mathematics (Moyon, 2022; Moyon, 2025).

organizations, one can mention the Scientific Instrument Commission of the IUHPST-DHST (International Union of History and Philosophy of Sciences and Technology – Division of History of Science and Technology) which hosts a dynamic group of researchers from various backgrounds on the place of historical scientific instruments in education; their annual meetings resulted in a collective book (Cavicchi & Heering, 2022); mathematical instruments from all periods have an important place in this group (Plantevin & Milici, 2022). Historical mathematical instruments constitute actually one way to introduce history of mathematics in education, and this line of research has been also explored for many years with a didactical perspective like in (Maschietto & Bartolini-Bussi, 2011) or (Poisard, 2016).

All these books and event constitute a very rich and various source of inspiration for the mathematics teachers willing to introduce historical perspectives in their classroom, but it had to be acknowledged that it remains a difficult step to take for teachers who most of time received no education in history of mathematics during their studies (Moyon, 2022). That's the reason why "Passerelles" was made for teachers, to give them activities and historical insights about them and an analysis of the tests with students. The book has met with a certain success as revealed by its second printing this year (2025) and a version in English to come this year (Moyon & *al*, forthcoming). This article is somehow the prequel of this series; in its first version[8] (in French), it presented the preliminary experiments of two of the nine groups which were about to contribute to the main book. Not only it shows the spirit of the common general approach and a flavor of the book, but for both concerned chapters, "The geometry of the notebooks of Leonardo da Vinci" and "The mechanization of calculation", it gives some insights which were referred to but not repeated in the chapters (either French or English). That's why the authors propose this article now (in English, to ensure broad accessibility), both introducing and founding a further reading of the main book, even though it can be read for itself. Both activities (and the whole project) were presented during a National Teacher Training Conference in Poitiers (France), June 2017[9].

Two domains of the French mathematics teaching program are addressed, "space and geometry" and "numbers and calculations", both with historical artefacts, a plate of drawings and mechanical calculating machines, from two different historical periods (respectively 15th and 17th century for the invention, 19th to 20th for the production). For each section, the reader will find an introduction that provides the historical and cultural context for the artefact understudy, the objectives and the description of the activity tested with students, their

[8] https://hal.science/hal-01781209
[9] https://irem.univ-poitiers.fr/colloque2017/

pedagogical and didactical issues and students' productions, together with their analysis.

1. DOING GEOMETRY WITH LEONARDO DA VINCI

1.1. Teaching aims

Our work[10] incorporates the teaching of plane geometry, approached through problem-solving activities based on the reproduction of figures.

> Reproducing various figures – both simple and complex – is a rich source of geometric problems. The level of difficulty can be adjusted depending on the figures to be reproduced and the tools available. Fundamental geometric concepts (lines, points, segments, right angles) are introduced through such problems. (Bulletin Officiel Spécial 11, 15/11/26, Cycle 2, Mathématiques, p. 83, our translation)

Moreover, the French school curriculum emphasizes the reproduction and construction of geometric figures. Among the objectives to be achieved for 8-11-year-old students, are the following:

> - Reproduce or construct a square, a rectangle, a triangle, a right triangle, or a circle, or combinations of these figures on any surface (graph paper, dotted paper, or plain paper), using a ruler, a set square, or a compass.
> - Construct a geometric figure composed of line segments, straight lines, common polygons, and circles. (Programme de mathématiques pour le cycle 3, https://www.education.gouv.fr/sites/default/files/ensel620_annexe2-v2.pdf, our translation)

Furthermore, our work is built on the didactic research of Marie-Jeanne Perrin-Glorian and her team[11]. In particular, in their work on the continuity of geometry teaching from ages 6 to 15, it is stated that:

> Doing geometry requires adopting a specific way of looking at figures […]: – one must be able to shift from seeing juxtaposed figures to seeing superimposed figures (and vice versa), by adding or erasing lines, etc.; – one must be able to perceive objects of various dimensions within the figures: surfaces, lines, points, and the relationships between them […]. (Perrin *et al.*, 2013, 17, our translation)

Understanding that a point is the intersection of two lines (not necessarily straight) is a key progressive goal for students aged from 8 to 11 years old. It is therefore essential to design rich problem-solving situations that gradually lead students to search for lines which, in turn, allow them to construct points. For instance, students must learn to see a vertex of a quadrilateral as the intersection of two sides, rather than merely as a corner of the figure (where the polygon is viewed only as a surface).

Indeed, if a student remains within a shape-based perception of geometry, they will be unable to create new lines to form points (such as endpoints or intersections) or to establish, for example, collinearity relationships.

[10] This section reports on the work of the group "Mathematics for 8-11-year-olds students through the History of Mathematics" from the IREM of Limoges, composed of Chantal Fourest, Jérôme Dufour (†), Véronique Lefranc, Marc Moyon, Valérie Rosier-David, and David Somdecoste.

[11] Let's cite, among others, (Duval *et al.*, 2005), (Keskessa *et al.*, 2007), (Offre *et al.*, 2007), (Perrin *et al.*, 2013) and more recenty (Perrin-Glorian, 2024).

The figure to be reproduced thus becomes a framework for exploring the geometric relationships that can be constructed. It offers an opportunity to experiment with what can be drawn and what is missing. Observing the figure becomes a crucial phase of analysis – that is, identifying the necessary conditions of the problem.

Students are therefore led to create points through the intersection of two lines, and in turn, these points themselves define lines. Two points can determine a segment, but also a circle (the center and a point on the circle). Two points can also define a ray or a straight line.

To work on figure reproduction, we chose to analyze certain figures from the famous *Codex Atlanticus* – a compilation of sketchbooks by Leonardo da Vinci, the renowned Renaissance artist, whom students aged from 8 to 11 years old are generally familiar with. Before presenting the pedagogical aspects of our work, let us first offer some historical background on Leonardo da Vinci and his *Codex*, which may be of interest to readers.

1.2. Leonardo da Vinci and the *Codex atlanticus*

Leonardo da Vinci was born in Vinci, Tuscany, and was largely self-taught, learning through books and discussions with scholars. He became an apprentice to Andrea del Verrocchio in Florence, where he joined the guild of painters and created his first machines.

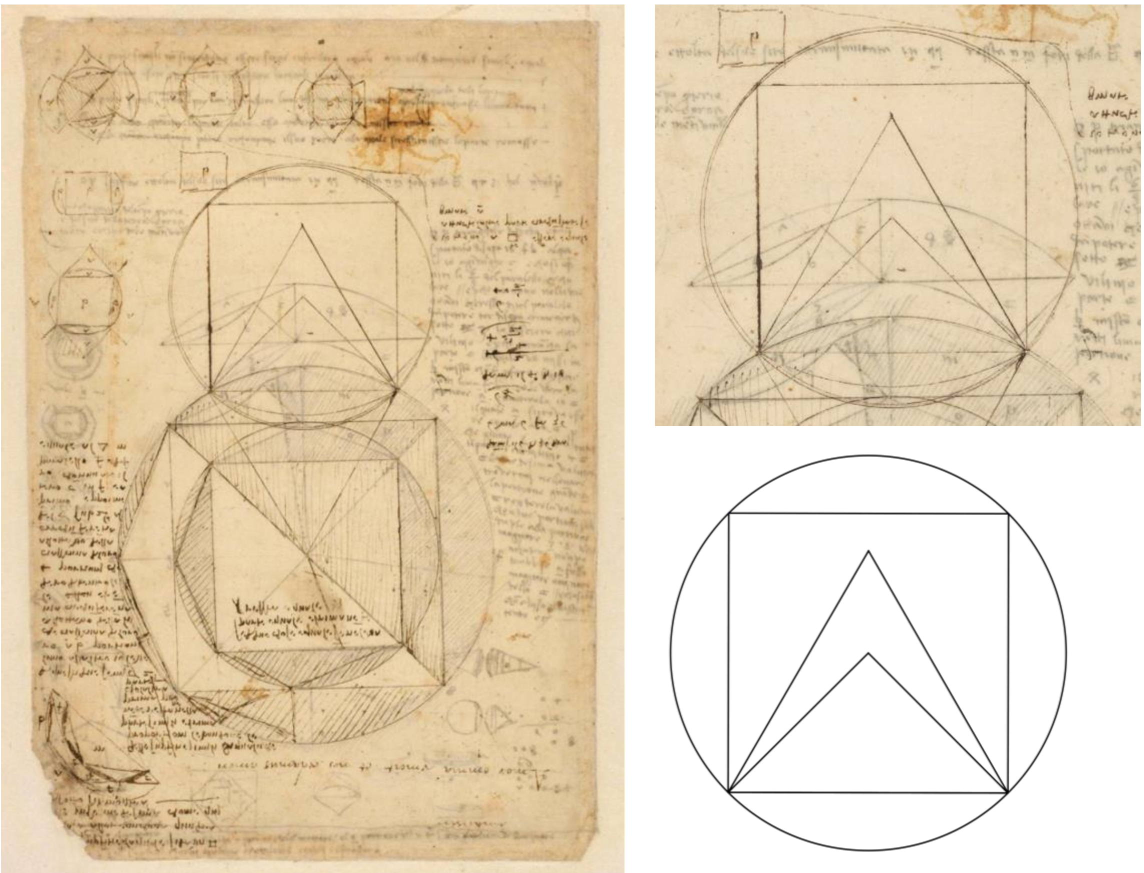
In 1482, he moved to Milan and was supported by Duke Ludovico Sforza. There, he explored painting, music, and engineering, designing machines and automata. By 1490, he was well established and began developing a strong interest in mathematics. In 1496, he met mathematician Luca Pacioli. After the French occupied Milan in 1499, Leonardo left with Pacioli and worked as a military engineer for Cesare Borgia.

Around 1506, he returned to Milan under French rule and began painting the Mona Lisa. After political upheaval in 1511, he moved to Rome and later, in 1516, to Amboise (France), invited by King Francis I. Despite suffering from paralysis, he continued working until his death in 1519.

Leonardo was a true polymath – painter, sculptor, inventor – whose work was grounded in careful observation of nature. He left behind numerous notes and drawings, many compiled in codices. One of the most famous, the *Codex Atlanticus*, contains over a thousand sheets and covers topics from flying machines to philosophy.

This codex inspired our team to create educational activities, particularly from folio 1040 recto, which addresses the ancient problem of the quadrature of the circle.

Figure 1 : *Codex atlanticus*, fol. 1040r



Source : (for educational use): https://codex-atlanticus.ambrosiana.it/#/Detail?detail=1040

1.3. Implementation of the pedagogical session

We designed a sequence of two sessions for students aged from 8 to 11 years old. Here, we provide an overview of the sessions and key elements of analysis. The two sessions enabled the introduction of several distinct phases: observation, construction, and the challenge of drafting a construction plan.

The observation phase is conducted collectively with 8-9-year-olds students. Its goal is to identify, based on the projection of Figure 2 (using a digital board or a simple chalk/whiteboard), all the basic shapes that make up the complex figure. The group discussion helps to create Tab. 1, which also serves as a summary of the activity.

Table 1: Geometrical figures and appropriated tools

Figure	**Definition or Property**	**Tools**
Circle	- Center - Radius	Compass
Square	- Side lengths - Right angles	Ruler + Set square
Equilateral triangle	3 sides of equal length	Ruler + Compass
Isosceles Triangle	2 sides of equal length	Ruler + Compass

Source: elaborated by the authors

It is also possible to give each student a copy of Leonardo's figure. However, when students have the figure directly in front of them, they tend to rely on measurement-based strategies rather than focusing on the relative positions of lines and the alignment of points. Given our goal, we believe it is preferable to impose the use of an ungraduated ruler. Length transfers (for the sides of the triangle) are then done with a compass: this represents an expert use of the compass that is explicitly targeted for students aged between 8 and11 years old.

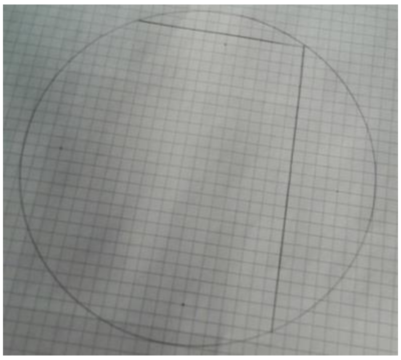
Figure 2: Use of measurement with graduated ruler

Source: Photo M. Moyon

For 10- to 11-year-old students, the observation phase is done individually within the working group, with each student having an original figure. This is an observation "with intention," meaning it should highlight not only the basic shapes but also the relationships between them, including alignment properties and points of intersection.

Observation is essential throughout the learning cycle (ages 8–11) because it provides students with all the elements needed to reproduce a figure and gradually allows them to move from a geometry of shapes to a geometry of points (Perrin *et al.*, 2013). Once the problem is clearly defined, students are asked to carry out the necessary constructions on plain white paper (A4 sheet). Students quickly realize that the order in which they construct the different basic shapes matters. For example, if they start with the circle – which is the most common approach since the circle encloses the figure – they then need to draw a square inscribed within that circle. 10- to 11-year-old students can achieve this by drawing two perpendicular diameters, which correspond to the square's diagonals. For 8- to 9-year-olds, this is difficult (if not impossible), since few know the property of the diagonals of a square[12]. After several attempts where the square is never quite square (see Fig. 4), students understand that it is better to start by drawing the square first and then draw its circumscribed circle.

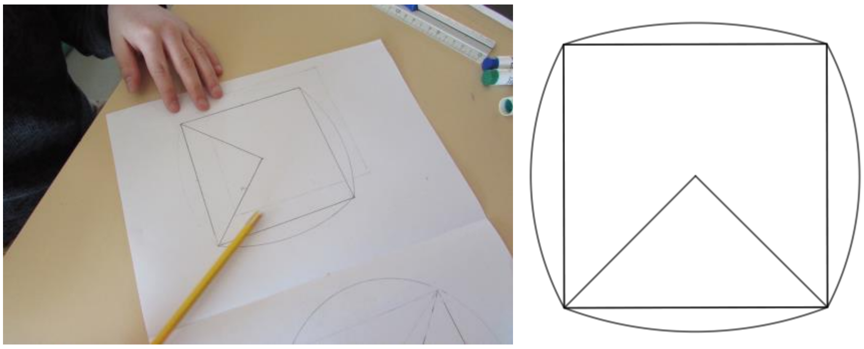
Figure 3: Difficult constructions of a square inscribed in a circle



Source: Photo M. Moyon

Students who have difficulty drawing a square on plain white paper are provided with graph paper (small squares). The permitted tools are a ruler, set square, and compass.

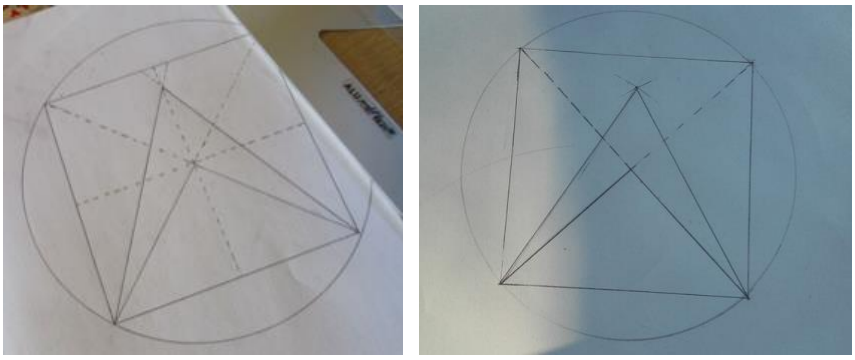
Figure 4: Example of a construction where the relationships between the figures are not respected: here, the circle is clearly not circumscribed around the square (the drawing lacks precision), but the student has drawn four arcs of circles, each having two vertices of the square as endpoints



Source: Photo M. Moyon

[12] To avoid this difficulty, for some students, an activity of "figure restoration" can be set up by providing, on a blank sheet, the square or two of its sides. The task is then to complete the figure (Perrin *et al.*, 2013, p. 26 and following).

Figure 5: Examples of figures where relationships—such as alignments, midpoints, and center—are indicated



Source: Photo M. Moyon

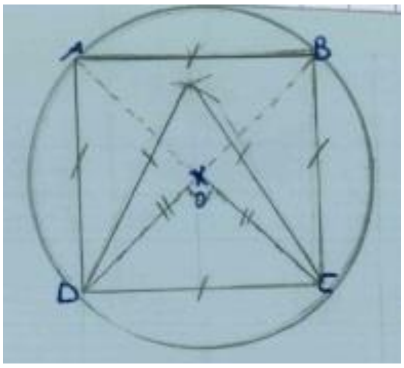
To successfully write a construction program for this complex figure, we guided the students through a construction narration. The students are then asked to write a text describing the different steps of their construction. Here are three examples of such narrations:

> **Example 1:** First, I drew a 4 cm square. Then, I drew an isosceles triangle and an equilateral triangle, both inside the square. Next, I drew a circle centered at the vertex of the equilateral triangle.
> **Example 2:** Draw a square with 5 cm sides. Draw the diagonals of the square. Then draw the circle passing through the vertices of the square. Draw an equilateral triangle with the base as one side of the square. Then do the same for an isosceles triangle.
> **Example 3:** First, I drew a square. Then, I drew the two diagonals of the square to find its center. This center will be the vertex of the right triangle whose other two vertices are the corners of the square. Next, I drew another triangle, this time equilateral, using the same two vertices as the right triangle. Finally, I drew a circle with a radius from the midpoint of the square to one of its vertices.

This step was carried out orally, as a group activity, with 8- to 9-year-old students. At all levels, the students generally respected the chronological order, using time connectors appropriately. They also mostly used the correct vocabulary. However, some confusion was noted between "midpoint" and "center," as well as between "midpoint" and "half."

Additionally, many students found it difficult to write their text without specifying two particular vertices of the square. Naming certain points on the figure then becomes very useful, if not essential.

Figure 6: Final figure with the names of key points useful for writing the construction program



Source: Photo M. Moyon

Here is, finally, the construction program developed collaboratively with 8- to 9-year-old students and tested in other classes of the same age group. The resulting figures are then compared with Leonardo da Vinci's original figure for validation.

> Draw a square ABCD. Draw the diagonals of the square; they intersect at point O. Draw the circle with center O passing through points A, B, C, and D. Draw triangle DOC. Draw the equilateral triangle with base DC.

In conclusion, the single figure on folio 1040 (recto) of Leonardo da Vinci's *Codex Atlanticus* offers several valuable opportunities for teaching geometry to students aged 8 to 11. On one hand, it grounds mathematical knowledge in the practice of a famous polymath and encourages interdisciplinary learning. Furthermore, it enables an approach to teaching geometry through problem-solving, with easily implemented differentiation strategies in the classroom. So, why miss out on it?

2. VARIOUS ASPECTS OF THE MECHANIZATION OF CALCULATIONS FOR WORKING ON NUMBERS AND ARITHMETIC OPERATIONS

The work[13] presented focuses on the numeration and calculation of operations, addition, subtraction, multiplication and division of the natural integers, using mechanical instruments that have partially automated them. We begin by presenting the machines used, placing them in their historical context. The mathematical objectives that can be pursued with these machines with students aged 8 to 11 are then outlined. Because of the multidisciplinary nature of the

[13] This section reports on the work of the research group 'Instruments in History and in the classroom' of the IREM of Brest. Véronique Fustec, Yohan Lefèbvre, Yann Le Thénaff from the middle school Collège St Pol Roux in Brest, Murielle Geslin from the Champ de foire school in Plougastel Daoulas, and Marc Le Pors from the Brest-Ville education district participated in the group's work between February and June 2017.

objects studied and the approach taken to use them, the activities can also achieve objectives in technology and history; this point is explained. The presentation of the experiment and some elements of analysis of two stages of the sequence conclude our presentation.

2.1. Calculation instruments used, context and operation

The mechanization of the four arithmetic operations began at the very beginning of the 17th century with the independent inventions of W. Schickard and B. Pascal[14]. Their machines both added whole numbers with automatic carryover. They are based on a new representation of whole numbers, with appropriately geared graduated wheels. For each order, there is a ten-tooth wheel[15], which makes turn the wheel on its left after one complete revolution. Pascal's machine, later known as the Pascaline, was the starting point for numerous attempts to improve the automatic calculation of addition and subtraction on the one hand, and multiplication (see Cargou *et al.*, 2025) and division on the other. The two machines we worked with represent an achievement in both these directions, almost three centuries later. They are a wheeled adder called Lightning calculator (Fig. 7) built in Michigan (USA) around 1930 according to a 1926 patent[16], and several Odhner-type multipliers built between 1900 and 1960 according to an original patent registered in Sweden in 1878, and then in France in 1890 and many other countries later[17].

Figure 7: The Lightning calculator adder

Source: Photo F. Plantevin

The Lightning calculator wheeled adder is an inexpensive, compact and fairly ergonomic office machine that was in production until 1960. Simple in appearance, its sophistication was due to the compactness of its mechanisms (which can be seen in the patent),

[14] Schickard’s invention was ahead of Pascal’s by some twenty years but could not play the same role in technical progress, as Pascal’s did, since the only example of the Schickard’s machine disappeared in a fire just after it was built and remained unknow till the 20th century.
[15] For the Pascal’s machines, it would be best adapted to speak of ten-rod wheels
[16] Patent US 1574249 by R.W. Hook ; it can be easily found on the internet by a search with the patent number; one can consult this well documented website for instance: https://www.jaapsch.net/mechcalc/lightning.htm#patents.
[17] Patents by W.T. Odhner in France and in the USA: FR207932, 1890-08-29, Paris, for 15 years and US 514725A, Washington, 1894-03-12; in between, patents were filed in Belgium, Luxembourg, Austria-Hungary, Norway, Sweden, Great Britain. See Appendix with note 20.

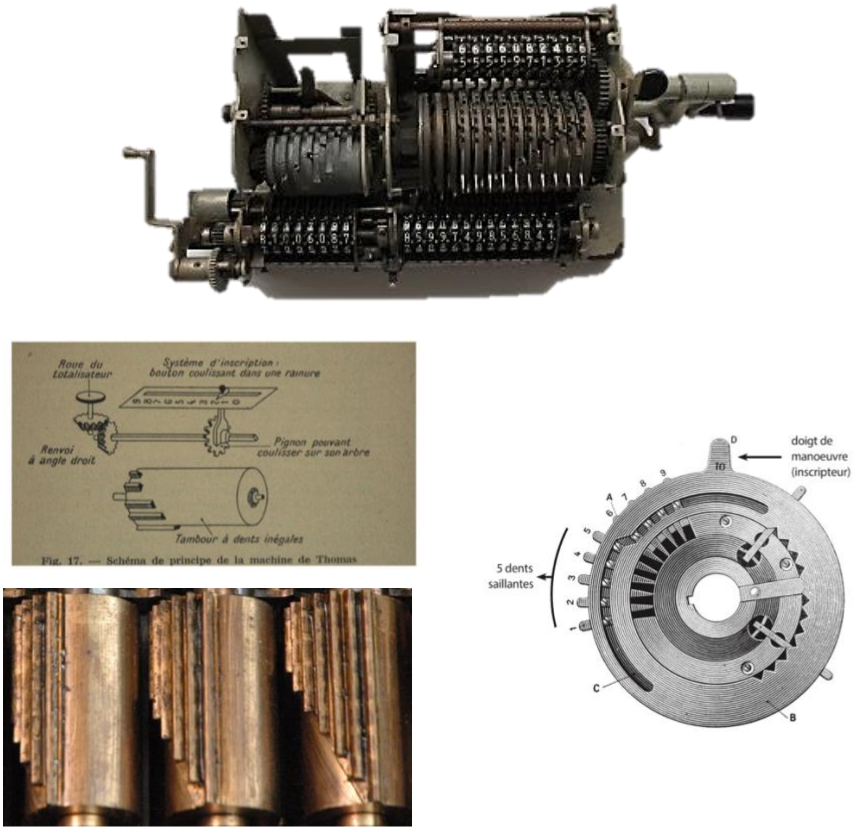
enclosed in a half-centimeter-thick casing, and to their reliability. Later models will also be able to perform subtractions, which is not the case with this one. To perform an addition, the operator enters the digits of each operand using the stylus. To enter 40, the stylus is inserted into the second wheel from the right, opposite graduation 4, and brought up against the stop - opposite 0 on the dial, turning clockwise while maintaining contact with the wheel. The number 4 appears in the window corresponding to this wheel.

Figure 8: Original Odhner multiplier

Source: Photo F. Plantevin

Odhner-type multipliers (i.e. the original machines and all their clones built on the same principle when the patent expired) perform multiplication by repeated additions. The Odhner machine was neither the first to do this, nor even the first to be manufactured industrially, but it was the most widely produced and widely used, right up to the 1970s. Multipliers are basically adding machines, but made more efficient by the addition of two parts: the driver and the carriage. The driver allows all the wheels to be driven at the same time, so that all the digits of both operands can be added at the same time; the carriage allows multiplication by ten by simply shifting the writing part of the machine in relation to the result-bearing part (called the totalizer). These two essential improvements were introduced by G.W. Leibniz in the late 17th century (Leibniz, 1710). Leibniz's driver uses a cylinder with grooves of unequal length (Fig. 9). This cylinder slides on its axis according to the inscriber's choice. The rotation of a toothed wheel, linked to the totalizer, geared to this cylinder therefore depends directly on its position on the cylinder, and therefore on the number entered on the inscriber. Two other types of driver were subsequently invented, including Odhner's variable-tooth trainer. In these machines, the inscriber drives a cam which draws out a variable number of teeth on a wheel. The rotation of this wheel, activated by the operator, will drive the mechanism only for the number of teeth selected by the position of the inscriber. The teeth of the ten Odhner drivers - one in each order - can be seen on the skin of Fig. 9.

Figure 9: Schematic diagram of the Leibniz driver and photo of three cylinders with stepped-drums. Explanatory diagram of an Odhner driver – A cut model of an Odhner multiplier (images taken from panels in the "Multiply!" exhibition)



Source: Photo F. Plantevin

So, to perform a multiplication with these machines, enter the digits of the first number and turn the crank a number of times equal to the units digit of the multiplier, then shift the carriage one notch to the right and turn the crank a number of times equal to the tens digit of the multiplier, and so on until you run out of digits. The result is read directly from the totalizer windows.

This family of instruments can be used to address a wide variety of issues in many fields. During workshops for the "Multiply!" exhibition[18], we were able to observe the interest of such work with school, middle-school and high-school students (Plantevin & Le Brusq, 2012). This gave us some initial ideas for use in mathematics classes.

[18] The exhibition retraced the history of calculations of the multiplication and in particular, its mechanization.

2.2. Mathematical objectives that can be pursued in the third learning (from 8 to 11 years old) around these machines

The mathematical objectives are multiple. On the one hand, there is reworking numeration through the representation of numbers offered by adding machines. To achieve this, we propose to analyze the operation of one of these machines and then build a prototype to enable students to really get to grips with the instrument. In this way, we can make the link between this representation and the operations of addition and subtraction, which are carried out by rotating the wheels in one direction or the other, provided the gearing enables automatic transfer of the carryover between two orders. By revisiting the previous learning cycle (from 6 to 8 years old) curriculum, one can understand the principle of mechanized addition (a technological issue), which is based on mathematical concepts, as mentioned above, and work on the representation and addition of large numbers (by "adding" the required number of wheels) and decimal numbers (by "adding" a physical marker representing the decimal point to the right of the wheel you want to be the units wheel). You can also work on the writing in line of the calculations performed, and thus on the associativity and commutativity of addition.

On the other hand, building on the first activity, we can then work on mechanized multiplication and Euclidean division. The implementation of multiplication by repeated additions by these machines relies on the associativity of addition and the distributivity of multiplication over addition, as well as on the base-10 writing of numbers since to calculate $65 \times 47 = 65 \times (4 \times 10 + 7)$, the machine actually performs

$$65 \times 47 = 65 \times 10 \times 4 + 65 \times 7 = 650 \times 4 + 65 \times 7 = \underbrace{650 + \cdots 650}_{4 \text{ times}} + \underbrace{65 + \cdots + 65}_{7 \text{ times}}$$

Understanding the instrument means being able to write this down, which makes it an interesting tool for the "Numbers and Calculations" chapter in the third teaching cycle.

Multiplication by 10 is achieved by physically shifting a part of the machine, and successive repeated additions are achieved by as many turns of the crank as the sum of the multiplier digits. A "simple" adder, like the Pascaline, would make

$$65 \times 47 = \underbrace{65 + \cdots + 65}_{47 \text{ times}}$$

i.e. forty-seven identical manipulations, each of which consists in entering two digits, i.e. ninety-four gestures, whereas with the multiplier, eleven turns of the crank are needed, plus two manipulations to enter the two digits of 65, i.e. thirteen manipulations. This small calculation of complexity clearly illustrates the evolution of calculating machines from adders to multipliers, and provides a simple but relevant illustration of the notion of complexity, so important in characterizing computer calculation algorithms.

With regard to division, let's follow the same reasoning as for multiplication and write in line the calculation performed by the machine when repeated subtractions from 15 to 47, for example, are carried out.

$47 - 15 - 15 - 15 = 2$, *i.e.* $47 - 3 \times 15 = 2$ or $47 = 3 \times 15 + 2$.

2 is the result of the operation, 3 is the number displayed by the revolution counter, and the machine beeps when the operator tries to subtract 15 from a smaller number: it thus makes it possible to materialize the quotient and remainder of Euclidean division and automates the stopping of the division, i.e. checks the positivity of the remainder.

2.3. Objectives in Sciences &Technology and History

Understanding how automated calculation is achieved, and then implementing it, can be tackled in both mathematics and technology, and can benefit both subjects. For this to happen, the mathematics used must be explained with the appropriate vocabulary and the required precision, and what has been learned in the joint activity or in each other's class, but which falls within the scope of one's own discipline, must be revisited and reinvested in each class. Let's take a closer look at the possible objectives for Technology and History.

The analysis and description of technical objects, and the design of "all or part of a technical object to translate a technological solution to a need", are at the heart of the "Materials and technical objects" theme of Science and Technology teaching program in third teaching cycle (*Bulletin Officiel de l'Éducation nationale*, 25, 2023-06-22)[19]. Both are covered by the proposed activity. The comparative study of different Odhner multiplier clones can be used to introduce the notion of technological evolution.

The industrialization of production methods is an important part of the history of these machines. It was Ch. X. Thomas de Colmar the first to improve Leibniz's machine in terms of functionality and reliability, but also so that it could be manufactured using industrial processes. This development work took him thirty-one years, between his first patent (1820) and the start of industrial production of his arithmometer (1851). Studying his career and his work on his machines is a pertinent illustration of the Industrial Revolution, which is part of the history curriculum for 9- to 10-year-old students (the second year of the third teaching cycle). We can also look at changes in production methods at some of the factories that made Odhner machine

[19] https://www.education.gouv.fr/sites/default/files/ensel101_annexe_okok.pdf

clones in the 20th century (e.g. Brunsviga, for which it is relatively easy to find a variety of resources). For middle school students (from 10 to 11 years old), the History curriculum cannot be directly linked to the proposed activity, but one of the aims throughout the third teaching cycle is to establish historical reference points in students.

The choice of this activity, which combines mathematics and technology, in addition to working on historical resources that may also involve history, is not just a reflection of the curiosity of the members of this project. We believe it is fruitful to develop the concrete aspects of mathematics to help students understand them better. We've noticed that some students show a kind of intuition or practical intelligence by manipulating and creating technical objects themselves. It seems to us that this talent is a possible springboard for learning related mathematics, and that it should be encouraged by well-chosen activities.

The activity we are proposing is clearly at the crossroads of three disciplines: it requires input from each discipline and builds knowledge in students in each discipline, in a way that is consistent with cycle programs. In middle school, and also at primary school, if the teacher has chosen to segment his or her teaching by discipline, the sequence as it is conceived should take place in the mathematics and technology slots; the case of history may be handled differently for the reasons already mentioned.

2.4. Introduction to experimentation and examples of sequence implementation

The experiment was carried out in three classes of 10- to 11-year-old students (thus last year of third cycle), with small variations. Below, we present an outline of the sequence and a few elements on two more specific points, based on student productions collected during the experiments (films or recordings of discussions during the research phases, photographs and films of prototypes produced). What is presented here also corresponds to what was done during the communication at the conference, which represents only a very small part of what was done in class. The documents distributed during the communication, correspond to some of the statements made in the classroom sessions; in this paper, they are referenced to as Appendices -although they are not annexed to it- but they are all made available on line[20].

The sequence is divided into two main stages, one dealing with adders and the other with multipliers. The sequence is as follows:

- understand how the addition of numbers is “automated” by studying the adder presented above (Appendix 1, statement for this session), conjecturing its inner workings and then

[20] This link: https://ubocloud.univ-brest.fr/s/TayWRKSRqg4mtxo leads the reader to a summary document with all available resources (referenced as appendices in this contribution).

building a wheeled adder that can add three-digit numbers (and whose result does not exceed three digits) with automatic carryover between each order. The idea for this prototype was inspired by the calculating machine proposed in the old *Bibliothèque de Travail magazine* (Pellissier, 1965). This part takes place in mathematics and technology, with co-leadership where possible. In math, the discovery of the adding machine and the beginning of the construction of the prototype represent three hours of work. In technology, the final stages of construction can be completed in two hours, depending on the desired finish and functionality.

- discover multipliers and their functionalities by performing additions, then repeated additions, identify where the operands can be read; specify how they differ from wheeled adders from a mathematical/algorithmic (operations performed) and technological (how they are performed) point of view: highlight the principle of the driver and the role of the carriage (linking it to the shift that appears in the calculation by hand of multiplication by a number with more than one digit); use them to perform multiplication calculations of fairly large numbers; proceed in the same way for subtraction and Euclidean division. This part was based on the statement proposed in Appendix 2[21] ("Working with a multiplier"); it was supplemented by live questions and work on patents (Appendix 3 and Appendix 4) and on exhibition catalogs such as the one for the exhibition already mentioned (Cargou *et al.*, 2025).

To illustrate how this can be implemented in the classroom, two points from the first stage of the sequence are detailed below.

2.5. Discovering the Lightning calculator adder

The aim of this session is to identify and name the various parts of the instrument, and to discuss and conjecture how it might work after some calculations have been carried out with it in front of the class. A document (Appendix 1) was developed to guide the analysis of the instrument and gather hypotheses as to its operation. It was used in two different ways in the classes, one distributed at the start of the session, the other after a lengthy discussion in the whole class. In both cases, it didn't quite fulfil the expected role; probably it was just too long.

Below are a few extracts from exchanges between the teacher and students during this discovery session, which began as a free discussion before using the document as a summary. Numerous calculations were made by the teacher, then by the students (Fig. 10); to continue the

[21] For all appendices, see note 20.

debate, the teacher asks the question: “We've done addition, even subtraction, so what do you think lie beneath?”

Figure 10: Calculations

Source: Photo F. Plantevin

They all agree without hesitation that there are gears[22], but it is not easy to go any further. Moreover, since the work of identifying the various parts of the instrument has not been done at this point, the students lack vocabulary, which we think prevents them from imagining how what they see might work. But not all of them, here is a discussion :

> **Student 1**: “I think that this [points to the units wheel] is a gear, but it's just that there are holes so you can turn them, and the teeth of the gear turn the other gears.”
> Much discussion about gears, wheels, then the teacher says: “What about the numbers in the story?”
> **Student 2**: “The numbers are on a kind of gear that has to turn according to the gear we're turning here [he points to the wheel inscribing the tens digit corresponding to the tens digit window in the result] so that it gives the number we want.”
> **The teacher** says: "Can you explain? I didn't quite understand."
> **Student 2**: “Actually, I think the numbers are written on a little wheel that's connected to the one you're supposed to turn so that you get the number you want afterwards.”
> **Student 3**: “Oh yes, for example, when you turn this one by 1 [shows a window of the result], it turns it just one notch.”

At the end of this discussion, and despite the fact that student 3 had reversed the logical order of things, almost all the essential parts of the machine had been identified: the wheel that turns under each dial has teeth (it wasn't said that there were 10), it drives a small wheel that displays the result corresponding to the number of notches the wheel has been turned by, the wheel that turns under the dial is also connected to the wheel under the left-hand dial so as to drive it after ten “notches” (it wasn't explicitly said at the time that this way, the carryover is done automatically). What remains to be done is to establish a precise vocabulary that will enable us to fully explain what we have understood. This synthesis phase is not described here.

2.6. Building a prototype adder

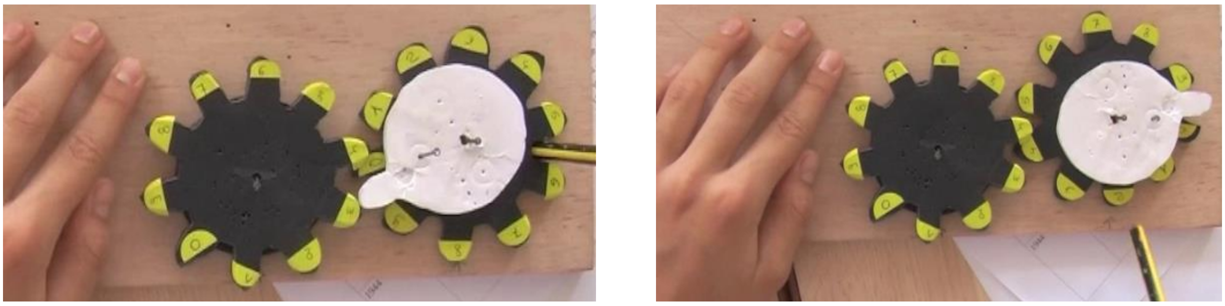
In the next session, students are asked to build their own wheel adder using the materials provided, in groups of three or four. The materials provided consist of a set of three ten-tooth

[22] The technology class on gears was held the previous week.

wheels, three one-tooth wheels with the same outside diameter, three disks with a diameter equal to the inside diameter of the tooth wheels, all nine made from foam core cardboard of the same thickness (not too soft to resist, not too hard to cut), three nails, a board, thumbtacks, stickers, a rectangular cover, and prepared but unfilled graduations. The first step is to make a functional two-wheel adder, then a three-wheel adder, and finally to finish off with the graduation and the cover. Here are some of the students' productions, with comments:

The photos in Figs. 11 to 13 illustrate the variety of solutions worked out by the students, all of whom at least met the beginning of the instructions for adding two and then three-digit numbers. Note the placement of the numbers on the wheels or on the fixed frame – which implies two different ways of working – and the stacking of the wheels on two or three levels to coordinate three wheels. Note the opposite direction of the graduations on two successive wheels, and the pins that hold the stacked wheels together. Let's take a closer look.

Figure 11: prototype 1 in the calculation of 7+5 and reading of the result



Source: Photo F. Plantevin

The photos in Fig. 11 show a student designer performing the calculation 7+5=12. You can see the 0-mark of each wheel on the board (indicated by a small arrow), the position of the wheel with a tooth responsible for the carryover between 9 and 0, and you can guess that the direction of rotation of the first wheel is counterclockwise. The graduation is on the wheels, and the student counts the elementary angles of rotation as he turns with his pencil.

Fig. 12 shows the prototype in the next phase of construction on three orders. It can be seen that the student has chosen to graduate his units wheel using a ten-tooth wheel (it's not a dial, however, as the graduation wheel is attached to the single-tooth wheel), so the student needs three levels of wheels for his prototype to work. Finally, we note that the direction of rotation of each wheel is indicated on the support, allowing the operands to be entered correctly. Note that the single-tooth units wheel has slipped from its correct position (compare with Fig. 14).

Figure 12: Prototype 1 with three wheels

Source: Photo F. Plantevin

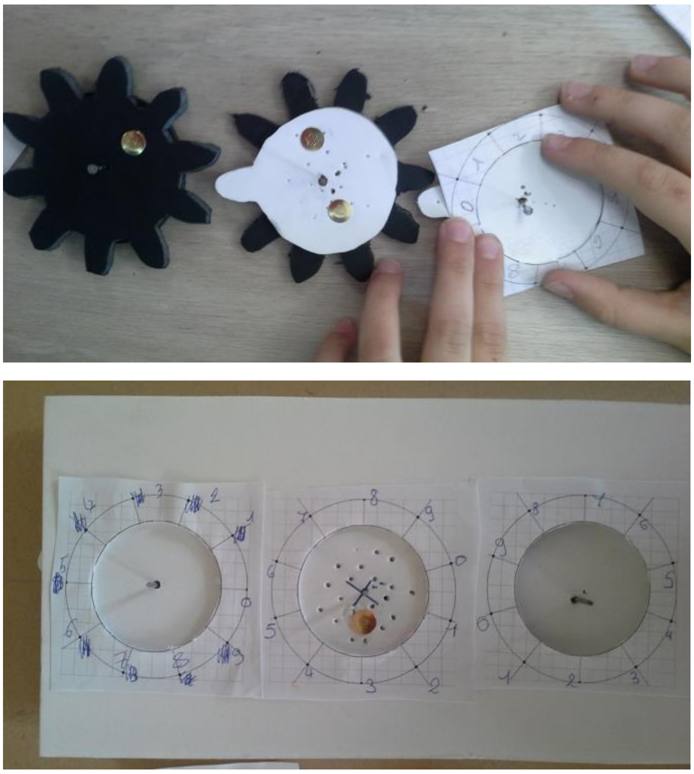
The prototypes shown in Fig. 13 feature graduations on the support (i.e. the frame). Prototype 2 features a kind of mixed graduation, in particular for the hundreds wheel (0 and 1): the 0 should also be indicated on the support.

Figure 13: Prototypes 2 and 3

Source: Photo F. Plantevin

In the design logic of these two and subsequent prototypes (Fig. 14), the origin of the graduation is given by the direction of rotation chosen for the first wheel (the units wheel with one tooth) and the “0 position” of this wheel, which must mesh with the tens wheel at each complete revolution. Here, the units wheel rotates in the trigonometric direction for these two prototypes, and the “position 0” of the unique tooth is therefore just after (with the orientation indicated) the point where it meshes with the teeth of the next wheel. However, there is no obligation to choose the “0 position” of the single tooth as the 0 graduation of a wheel (as is done on prototype 4), but it is essential to locate this famous position on the wheel in order to graduate correctly. To perform the calculation 7+5=12 with such a graduation, the manipulator plants a pointer (here a nail or compass point) opposite graduation 0 and rotates the wheel keeping the pointer in the wheel until it is opposite 7, then plants the pointer opposite graduation 0 and rotates the wheel keeping the pointer in the wheel until it is opposite graduation 5. The result is read off the scale opposite the 0 position mark (on the wheel).

Figure 14: Prototypes 4 and 5 in the last stage of construction – Fitting the graduated frame



Source: Photo F. Plantevin

The last part of the activity consists in building a graduated cover to use the prototype as a real adder. Fig. 14 shows two stages of this construction; the graduation of the third wheel of prototype 5, which had been graduated in the same direction as the previous wheel, can be seen crossed out, whereas it now turns (and therefore counts) in the opposite direction.

As can be seen from the study of these prototypes, this activity has a number of fairly subtle points, requiring a back-and-forth between experimentation, knowledge of the operation and a deep understanding of positional decimal numeration; it enables each student to seek a solution that is both personal (see the diversity of the prototypes) and universal (as it has to meet precise specifications); finally, it leads the students to debate and examine each other's ideas within their group, and then to explain the chosen solution to the teachers. For the classroom sessions, as for the communication at the conference, we were able to borrow instruments that we could manipulate. It is obviously preferable to do calculations with the machines, so as to get to grips with how they work. But having them available can be difficult (even though they are still fairly widespread these days), so we thought about using dedicated videos to accompany the whole sequence. This is what we did next, that you can find in the final project books (Moyon *et al*., 2018 and 2025), in French and in English.

At the beginning of the work with mechanical calculators, adders like the Lightning calculator were just considered as a step to reach an understanding of the more complicated multipliers, but finally it appeared that the sequence where students built their own adding prototype was very interesting by itself and really within the reach of young students (very beginning of the third cycle). The group "Instruments in History and in the classroom" focused its work on this activity, being reinforced with a primary school teacher, Priscilla Guéna (†), École du Vieux Poirier in Goulven (France). It led to resources for professional development days for in-service primary school teachers; material and some feedback of these experiences are presented in (Plantevin, 2024).

CONCLUSION

In the present contribution, mathematics teaching material which integrate historical perspective was presented; tested with students, its presentation is augmented with detailed historical, pedagogical and didactical insights. By focusing on the main obstacles and difficulties faced by students based on some of their own work and teaching assignments, we showed our intention to write firstly for the teachers, so that they could make these activities on their own and actually implement them in their classroom. For this reason, this article (and the book to come) could clearly be used for teacher training too; that's the reason why both activities (and the whole project) were originally presented during a National Teacher Training Conference (in Poitiers 2017).

Although focused on the discipline of mathematics, the activities are also considered with numerous interdisciplinary crossovers (with French, History and Geography, or Technology). They provide the opportunity to work explicitly many skills that are central in mathematics: researching, modeling, representing, calculating, reasoning, communicating, but also some of other discipline's skills like getting time markers, understanding a document namely, a historical one, studying and conceiving a technical object ... Finally, we believe, as the French official instructions precise[23], that more generally: "Putting certain knowledge into

[23] *Bulletin Officiel Spécial* 11, 26/11/15, Cycle 3, Mathématiques, p. 198, our translation.

historical perspective (positional numeration, the appearance of decimal numbers, the metric system, etc.) helps enrich students' scientific culture.”

REFERENCES

Barbin É., Guichard J.-P., Moyon M., Guyot P., Morice-Singh C., Métin F., Buhler M., Tournès D., Chorlay R. & Hamon Gérard (2018). *Let history into the mathematics classroom*. Springer. https://doi.org/10.1007/978-3-319-57150-8

Cargou, C., Cargou, M.-P. & Plantevin, F. (2025). *Multiply! Multiplication calculation instruments. 200 years of technical and industrial genius*. IREM de Brest.

Cavicchi E. & Heering P. (2021). *Historical scientific instruments in contemporary education.* Brill. https://doi.org/10.1163/9789004499676

Clark K., Kjeldsen T., Schorcht S., Tzanakis C. & Wang X. (2016) History of mathematics in mathematics education : recent developments. In L. Radford, F. Furinghetti & T. Hausberger (Eds). *Proceedings of the 2016 ICME satellite meeting of the international study group on the relations between the history and pedagogy of mathematics* (pp. 135-179). IREM de Montpellier.

Plantevin F & Milici P. (2021). Chapter 11: Historical instruments, education, and do-it-yourself in the cabinet of curiosity of Brest, France: university experiences in mathematics. In E. Cavicchi & P. Heering (Eds). *Historical scientific instruments in contemporary education* (pp. 209-225). Brill. https://doi.org/10.1163/9789004499676_013

Duval R., Godin M. & Perrin-Glorian M.-J. (2005). Reproduction de figures à l’école élémentaire. In C. Castela C., C. Houdement (Eds) *Actes du séminaire national de didactique des mathématiques* (pp. 5-89). ARDM, IREM Paris 7.

Fauvel J. & Van Maanen J. (2000) *History in mathematics education : the icmi study*. Kluwer Academic Publishers.

Katz V. & Tzanakis C. (2011). Recent developments on introducing a historical dimension in mathematics education. Mathematical Association of America.

Keskessa B., Perrin-Glorian M.-J.. & Delplace J.-R. (2007). Géométrie plane et figures au cycle 3. Une démarche pour élaborer des situations visant à favoriser une mobilité du regard sur des figures de géométrie. *Grand N*, 79, 33-60. https://irem.univ-grenoble-alpes.fr/medias/fichier/79n3_1554797089105-pdf

Leibniz G. W. (1710). Brevis descriptio Machinæ. In *Miscellanea Berolinensia* (317-319).

Maschietto M., Bartolini-Bussi M. G., (2011). Mathematical machines : from history to mathematics classroom. . In O. Zaslavsky & P. Sullivan (Eds). *Constructing knowledge for teaching secondary mathematics. Mathematics teacher education* (vol. 6, pp. 227-245). Springer. https://doi.org/10.1007/978-0-387-09812-8_14

Moyon M. (2022). Desire of teachers and realities in textbooks: dealing with history of mathematics in the new French curriculum and its impact on teacher training, *ZDM – Mathematics Education*, 7(54), 1613-1630. https//doi.org/10.1007/s11858-022-01427-6

Moyon M. (2025). *O liber abbaci de fibonacci: das matemáticas medievais para o ensino atual*. LF editorial.

Moyon M. (2025). (Re)découvrir les mathématiques médiévales : l'héritage de Fibonacci pour l'enseignement des mathématiques d'aujourd'hui, *Petit x* 122, 5-29. https://irem.univ-grenoble-alpes.fr/medias/fichier/122x1_1750603533701-pdf

Moyon M. & Tournès, D. (dir.) (2018 1st printing, 2025 2nd printing). *Passerelles : enseigner les mathématiques par leur histoire au cycle 3*. ARPEME.

Moyon M., Tournès, D. & Lawrence, S. (dir.) (to appear). *Teaching mathematics through historically-based activities – experiments in French classrooms*. Springer-Verlag.

Offre, B., Perrin-Glorian, M.-J. & Verbaere, O. (2006). Usage des instruments et des propriétés géométriques en fin de CM2. *Petit x* 72, 6-39. https://irem.univ-grenoble-alpes.fr/medias/fichier/77n2_1554800347159-pdf

Pélissier, M. (1965). Construis une machine à calculer (addition et soustraction), *Supplément au n°612 de Bibliothèque de Travail*, 189.

Perrin-Glorian, M.-J., Mathé, A.-C. & Leclercq, R. (2013). Comment peut-on penser la continuité de l'enseignement de la géométrie de 6 à 15 ans ? Le jeu sur les supports et les instruments. *Repères-IREM* 90, 5-41. https://irem.univ-grenoble-alpes.fr/medias/fichier/iwr13003_1704892231717-pdf

Perrin-Glorian, M.-J. (2024). Teach plane geometry in coherence from 6 to 15 years. REMATEC 19(48). Doi: 10.37084/rematec.1980-3141.2024.n48.e2024001.id588.

Plantevin F., Le Brusq C. (2013). Exploitation pédagogique d'une exposition d'instruments de calcul. In *Actes du XXXIXe colloque COPIRELEM Quimper*, pp. 152-180. IREM de brest.

Plantevin, F. (2024). Roues dentées et engrenages, nombres et opérations : allers et retours. *Repères-IREM* 136, 5-30. https://irem.univ-grenoble-alpes.fr/medias/fichier/136r1_1736973399796-pdf

Poisard C. (Ed.) (2016). Les ressources virtuelles et matérielles en mathématiques : des instruments pour travailler en classe sur le nombre, la numération et le calcul. *Mathématice* 51 special issue. http://revue.sesamath.net/spip.php?rubrique135